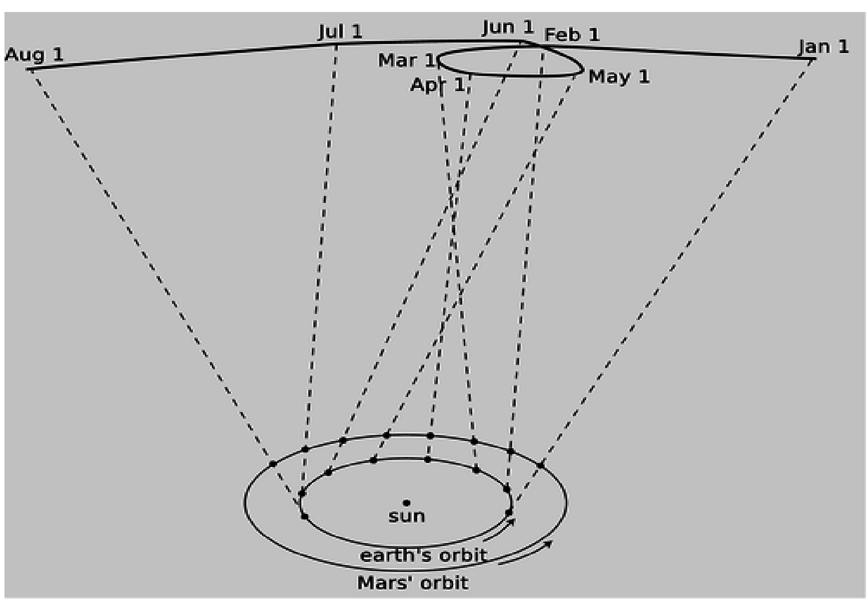
# GRAVITATIONAL PHENOMENA

Cruise your radio dial today and try to find any popular song that would have been imaginable without Louis Armstrong. By introducing solo improvisation into jazz, Armstrong took apart the jigsaw puzzle of popular music and fit the pieces back together in a different way. In the same way, Newton reassembled our view of the universe. Consider the titles of some recent physics books written for the general reader: **The God Particle**, **Dreams of a Final Theory**. When the subatomic particle called the neutrino was recently proven for the first time to have mass, specialists in cosmology began discussing seriously what effect this would have on calculations of the evolution of the universe from the Big Bang to its present state. Without the English physicist Isaac Newton, such attempts at universal understanding would not merely have seemed ambitious, they simply would not have occurred to anyone.

This section is about Newton's theory of gravity, which he used to explain the motion of the planets as they orbited the sun. Newton tosses off a general treatment of motion in the first 20 pages of his **Mathematical Principles of Natural Philosophy**, and then spends the next 130 discussing the motion of the planets. Clearly he saw this as the crucial scientific focus of his work. Why? Because in it he showed that the same laws of nature applied to the heavens as to the earth, and that the gravitational interaction that made an apple fall was the same as the as the one that kept the earth's motion from carrying it away from the sun.

## Kepler's laws

Newton wouldn't have been able to figure out *why* the planets move the way they do if it hadn't been for the astronomer Tycho Brahe (1546-1601) and his protege Johannes Kepler (1571-1630), who together came up with the first simple and accurate description of *how* the planets actually do move. The difficulty of their task is suggested by the figure below, which shows how the relatively simple orbital motions of the earth and Mars combine so that as seen from earth Mars appears to be staggering in loops like a drunken sailor.

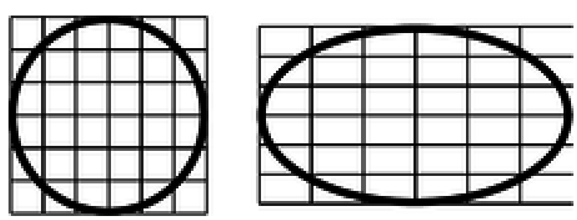



d / As the earth and Mars revolve around the sun at different rates, the combined effect of their motions makes Mars appear to trace a strange, looped path across the background of the distant stars.

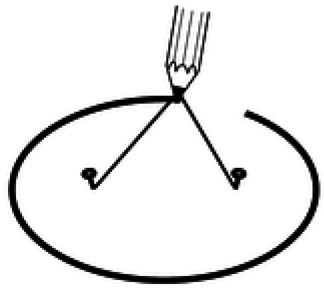
Brahe, the last of the great naked-eye astronomers, collected extensive data on the motions of the planets over a period of many years, taking the giant step from the previous observations' accuracy of about 10 minutes of arc (10/60 of a degree) to an unprecedented 1 minute. The quality of his work is all the more remarkable considering that his observatory consisted of four giant brass protractors mounted upright in his castle in Denmark. Four different observers would simultaneously measure the position of a planet in order to check for mistakes and reduce random errors.

With Brahe's death, it fell to his former assistant Kepler to try to make some sense out of the volumes of data. After 900 pages of calculations and many false starts and dead-end ideas, Kepler finally synthesized the data into the following three laws:

**Kepler's elliptical orbit law:** The planets orbit the sun in elliptical orbits with the sun at one focus.

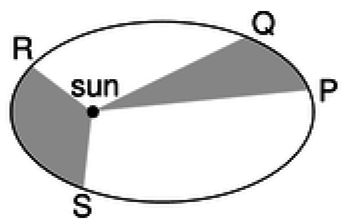


a / An ellipse is circle that has been distorted by shrinking and stretching along perpendicular axes.



b / An ellipse can be constructed by tying a string to two pins and drawing like this with a pencil stretching the string taut. Each pin constitutes one focus of the ellipse.

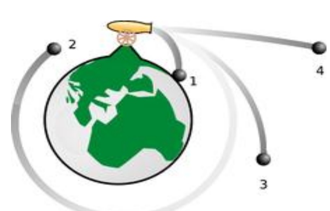
**Kepler's equal-area law:** The line connecting a planet to the sun sweeps out equal areas in equal amounts of time.




c / If the time interval taken by the planet to move from P to Q is equal to the time interval from R to S, then according to Kepler's equal-area law, the two shaded areas are equal. The planet is moving faster during time interval RS than it was during PQ, because gravitational energy has been transformed into kinetic energy.

**Kepler's law of periods:** The time required for a planet to orbit the sun, called its period, T, is proportional to the long axis of the ellipse raised to the 3/2 power. The constant of proportionality is the same for all the planets. Although the planets' orbits are ellipses rather than circles, most are very close to being circular. The earth's orbit, for instance, is only flattened by 1.7% relative to a circle. In the special case of a planet in a circular orbit, the two foci (plural of "focus") coincide at the center of the circle, and Kepler's elliptical orbit law thus says that the circle is centered on the sun. The equal-area law implies that a planet in a circular orbit moves around the sun with constant speed. For a circular orbit, the law of periods then amounts to a statement that the time for one orbit is proportional to $r^{3/2}$, where r is the radius. If all the planets were moving in their orbits at the same speed, then the time for one orbit would simply depend on the circumference of the circle, so it would only be proportional to r to the first power. The more drastic dependence on $r^{3/2}$ means that the outer planets must be moving more slowly than the inner planets. Our main focus in this section will be to use the law of periods to deduce the general equation for gravitational energy. The equal-area law turns out to be a statement on conservation of angular momentum, which is discussed in chapter 4. We'll demonstrate the elliptical orbit law numerically in chapter 3, and analytically in chapter 4.2.3.2 Circular orbits Kepler's laws say that planets move along elliptical paths (with circles as a special case), which would seem to contradict the proof on page 90 that objects moving under the influence of gravity have parabolic trajectories. Kepler was right. The parabolic path was really only an approximation, based on the assumption that the gravitational field is constant, and that vertical lines are all parallel. In figure e, trajectory 1 is an ellipse, but it gets chopped off when the cannonball hits the earth, and the small piece of it that is above ground is nearly indistinguishable from a parabola. Our goal is to connect the previous calculation of parabolic trajectories, $y=(g/2v^2)x^2$, with Kepler's data for planets orbiting the sun in nearly circular orbits. Let's start by thinking in terms of an orbit that circles the earth, like orbit 2 in figure e. It's more natural now to choose a coordinate system with its origin at the center of the earth, so the parabolic approximation becomes $y=r-(g/2v^2)x^2$, where r is the distance from the center of the earth. For small values of x, i.e., when the cannonball hasn't traveled very far from the muzzle of the gun, the parabola is still a good approximation to the actual circular orbit, defined by the Pythagorean theorem, $r^2=x^2+y^2$, or $y=r\sqrt{1-x^2/r^2}$. For small values of x, we can use the approximation $\sqrt{1+\epsilon} \approx 1+ \epsilon/2$ to find $y \approx r - (1/2r)\, x^2$. Setting this equal to the equation of the parabola, we have $g/2v^2=(1/2r)$, or

$$v=\sqrt{gr} \text{[condition for a circular orbit].}$$




e / A cannon fires cannonballs at different velocities, from the top of an imaginary mountain that rises above the earth's atmosphere. This is almost the same as a figure Newton included in his **Mathematical Principles**

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