FARADAY’S INDUCTION LAW

1) A stationary circuit in a time-varying field:
   (Transformer emf)

It is well known that current carrying conductor produces a magnetic field, the inverse of this is also true.

A magnetic field can produce a current in a closed circuit but the magnetic flux linkage by the circuit must be changing with the time.
Refering to the above figure the magnetic field due to the induced current opposes the magnet moment, thus moving the magnet up and down produces AC current.

If the loop is open circuited an emf apperas at its terminals. i.e.

\[ emf = V_{emf} = - \frac{d\psi}{dt} \]  \hspace{1cm} \text{(1)}
Where $\psi$ is the total flux linkage. If the loop has $N$ turns:

\[
\nu_{emf} = -N \frac{d\psi}{dt}
\]

The emf induced in the loop is equal to the emf producing field $\bar{E}$, thus:

\[
\nu_{emf} = \oint \bar{E}.d\bar{l}
\]

The total flux through a circuit is equal to the normal component of the flux density $\bar{B}$ over the surface bounded by the circuit, i.e:

\[
\psi = \iint_S \bar{B}.d\bar{s} \quad \ldots \ldots \ldots \ldots \ldots (2)
\]

Where $S$ is the surface bounded by the closed circuit.
From (1) and (2)
\[ v_{emf} = -N \frac{d}{dt} \int_A \overrightarrow{B} \cdot d\overrightarrow{s} \]

\( B \) : flux in Tesla (Wb/m\(^2\)).

\( t \): time in seconds.

\( d\overrightarrow{s} \): surface element in m\(^2\).

If the circuit is stationary or fixed, then

\[ v_{emf} = -N \int_A \frac{\partial \overrightarrow{B}}{\partial t} \cdot d\overrightarrow{s} \]

2) Moving Conductor in a Magnetic Field:
(Motional emf)

Lorentz Force \( \overrightarrow{F} \) on a particle of charge \( Q \) moving with a velocity \( \overrightarrow{v} \) in a magnetic field \( \overrightarrow{B} \) is:

\[ \overrightarrow{F} = Q(\overrightarrow{u} \times \overrightarrow{B}) \]
\[ \frac{\vec{F}}{Q} = (\vec{u} \times \vec{B}) = \vec{E} \]

\[ v_{emf} = \int_{1}^{2} \vec{E}.d\vec{l} \]

\[ v_{emf} = \int_{1}^{2} \vec{u}X\vec{B}.d\vec{l} \]

For a closed circuit moving in a magnetic field with velocity \( \vec{u} \):

\[ v_{emf} = \int_{1}^{2} \vec{u}X\vec{B}.d\vec{l} \]
3) General Case of Induction

The total emf induced in a closed circuit due to the time varying field and moving by velocity $\bar{u}$ is given by:

$$\bar{V}_{emf} = \int \bar{u} \times \bar{B} \cdot d\bar{l} - \iint_{S} \frac{d\bar{B}}{dt} \cdot d\bar{s}$$

emf due to motion                 emf due to time varying $\bar{B}$

Sign Convention (Lenz’s Law):

1) Transformer emf

a) Increasing $\bar{B}$, increases $\psi_i$.

b) The induced current produces a flux $\psi_i$ trying to compensate for the increase of the flux due to $\bar{B}$. 
c) $E_e$ is the induced potential:

$$V = \frac{1}{2} \int \vec{E}_e \cdot d\vec{l} = -A \frac{d\vec{B}}{dt}$$

2) Motional emf

a) $\Psi_i$ increases due to the movement of the bar.

b) The induced current produces a flux $\Psi_i$ trying to compensate for the increase of the flux due to $\vec{B}$.

c) $v = \frac{1}{2} \int \vec{E}_e \cdot d\vec{l} = \vec{u} \times \vec{B} \cdot l = -B \frac{\partial A}{\partial t}$
Where $A$ is the area of the circuit.