Evolutionary Design on-line Sliding Fuzzy Gain Scheduling Sliding Mode Algorithm: 
Applied to Internal Combustion Engine

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Abstract
Refer to this research, a position on-line fuzzy sliding gain scheduling sliding mode control (AFSGSMC) design and application to internal combustion engine has proposed in order to design high performance nonlinear controller in the presence of uncertainties and external disturbance. Even though, sliding mode controller (SMC) is used in wide range areas but it has the following disadvantages: chattering and equivalent dynamic formulation. The fuzzy on-line tuneable sliding function in fuzzy sliding mode controller is based on Mamdani’s fuzzy inference system (FIS) and it has one input and one output. The input represents the function between sliding function, error and the rate of error. The outputs represent the dynamic estimator to estimate the nonlinear dynamic equivalent in supervisory fuzzy sliding mode algorithm. The fuzzy sliding mode methodology is on-line tune the sliding function based on self tuning coefficient methodology. The performance of the AFSGSMC is validated through comparison with previously developed IC engine controller based on sliding mode control theory (SMC). Simulation results signify good performance of fuel ratio in presence of uncertainty and external disturbance.

Keywords: internal combustion engine, sliding mode controller, sliding mode fuzzy controller, on-line sliding fuzzy gain scheduling sliding mode control.

1. Introduction and Motivation

Modeling of an entire IC engine is a very important and complicated process because internal combustion engines are nonlinear, multi inputs-multi outputs (MIMO) and time variant. There have been several engine controller designs over the previous years in which the main goal is to improve the efficiency and exhaust emissions of the automotive engine [1-4]. Specific applications of air to fuel (A/F) ratio control based on observer measurements in the intake manifold were developed by Benninger in 1991 [5]. Another approach was to base the observer on measurements of exhaust gases measured by the oxygen sensor and on the throttle position, which was researched by Onder [6]. These observer ideas used linear observer theory. Hedrick also used the measurements of the oxygen sensor to develop a nonlinear, sliding mode approach to control the A/F ratio [7]. All of the previous control strategies were applied to engines that used only port fuel injections, where fuel was injected in the intake manifold. Current production A/F ratio controllers use closed loop feedback and feed forward control to achieve the desired stoichiometric mixture. These controllers use measurements from the oxygen sensor to control the desired amount of fuel that should be injected over the next engine cycle and have been able to control the A/F very well [8].

However, one of the important challenge in control algorithms is to have linear controller behavior for easy implementation of nonlinear (e.g., IC engine) systems but these algorithms however have some limitation such as controller working area must to be near system operating point and this adjustment is very difficult.
especially when the system dynamic parameters have large variations and when the system has hard nonlinearities [1, 6, 9]. Most of IC engine which work in industry are usually controlled by linear PID controllers. But the IC engine dynamic functions are, nonlinear with strong coupling, structure and unstructured uncertainty and Multi-Inputs Multi-Outputs (MIMO) which, design linear controller is very difficult especially if the velocity and acceleration of IC engine be high [2]. To eliminate above problems in physical systems most of control researcher go toward to select nonlinear robust controller. Sliding mode controller (SMC) is the powerful nonlinear controllers in systems which are used to solve control problems [10]. Conversely, this controller is used in different applications with good performance; sliding mode controller has chattering or oscillation challenge in certain or uncertain system [11-17]. In order to solve the chattering in the systems output, boundary layer method should be applied so beginning able to recommended model in the main motivation which in this method the basic idea is replace the discontinuous method by saturation (linear) method with small neighborhood of the switching surface [11-17]. Slotine and Sastry have introduced boundary layer method instead of discontinuous method to reduce the chattering[18]. Estimated uncertainty method is used in term of uncertainty estimator to compensation of the system uncertainties. It has been used to solve the chattering phenomenon and also nonlinear equivalent dynamic. The applications of artificial intelligence, neural networks and fuzzy logic on estimated uncertainty method have been reported in [19-22]. Wu et al. [23] have proposed a simple fuzzy estimator controller beside the discontinuous and equivalent control terms to reduce the chattering.

In different dynamic parameters systems or in uncertain parameter systems (e.g., IC engine) which to have good response need to be training, adaptive gain scheduling methodology is used. In this research in order to solve disturbance rejection and uncertainty dynamic parameter, adaptive fuzzy sliding gain scheduling method is applied to sliding mode controller. F Y Hsu et al. [24]have presented adaptive fuzzy sliding mode control which can update fuzzy rules to compensate nonlinear parameters and guarantee the stability robot manipulator controller. This paper is organized as follows: In section 2, main subject of engine operating cycle and detail dynamic formulation of modeling in IC engine are presented. Detail of proposed adaptive fuzzy sliding gain scheduling algorithm sliding mode controller is presented in section 3. In section 4, the simulation result is presented and finally in section 5, the conclusion is presented.


In developing a valid engine model, the concept of the combustion process, abnormal combustion and cylinder pressure must be understood. The combustion process is relatively simple and it begins with fuel and air being mixed together in the intake manifold and cylinder. This air-fuel mixture is trapped inside cylinder after the intake valve(s) is closed and then gets compressed [25]. When the air-fuel mixture is compressed it causes the pressure and temperature to increase inside the cylinder. In abnormal combustion, the cylinder pressure and temperature can rise so rapidly that it can spontaneously ignite the air-fuel mixture causing high frequency cylinder pressure oscillations. These oscillations cause the metal cylinders to produce sharp noises called knock, which it caused to abnormal combustion. The pressure in the cylinder is a very important physical parameter that can be analyzed from the combustion process. Since cylinder pressure is very important to the combustion event and the engine cycle in spark ignition engines, the development of a model that produces the cylinder pressure for each crank angle degree is necessary. A cylinder pressure model that calculates the total cylinder pressure over 720 crank angle degrees was created based upon the following formulation [25-27]:

\[
P_{\text{cyl}}(\theta) = P_{m}(\theta) + P_{\text{net}}(\theta)
\]

(1)

where \( P_{\text{cyl}}(\theta) \) is pressure in cylinder, \( P_{m}(\theta) \) is Wiebe function, and \( P_{\text{net}}(\theta) \) is motoring pressure of a cylinder. Air fuel ratio is the mass ratio of air and fuel trapped inside the cylinder before combustion starts. Mathematically it is the mass of the air divided by the mass of the fuel as shown in the equation below:

\[
\text{Air to Fuel} = \frac{m_{\text{air}}}{m_{\text{fuel}}}
\]

(2)

If the ratio is too high or low, it can be adjusted by adding or reducing the amount of fuel per engine cycle that is injected into the cylinder. The fuel ratio can be used to determine which fuel system should have a larger impact on how much fuel is injected into the cylinder. Since a direct fuel injector has immediate injection of its fuel with significant charge cooling effect, it can have a quicker response to the desired amount of fuel that is needed by an engine [27].

3. Design Proposed Fuzzy Sliding Gain Scheduling Sliding Mode Algorithm
IC engines are one of the highly nonlinear and uncertain systems which caused to needed to robust controller. This section provides introducing the formulation of sliding mode controller to IC engine based on [11-17]. Consider a nonlinear single input dynamic system of the form [11-12]:

\[ x^{(n)} = f(x) + b(x)u \]  

(3)

Where \( u \) is the vector of control input, \( x^{(n)} \) is the \( n^{th} \) derivation of \( x \), \( x = [x, \dot{x}, \ddot{x}, ..., x^{(n-1)}]^T \) is the state vector, \( f(x) \) is unknown or uncertainty, and \( b(x) \) is of known sign function. The control problem is to track the desired state; \( x_d = [x_d, \dot{x}_d, \ddot{x}_d, ..., x_d^{(n-1)}]^T \), and have an acceptable error which is given by:

\[ \ddot{x} = x - x_d = [\ddot{x}, ..., \ddot{x}^{(n-1)}]^T \]  

(4)

A time-varying sliding surface \( s(x, t) \) is given by the following equation:

\[ s(x, t) = (\frac{d}{dt} + \lambda)\ddot{x} = 0 \]  

(5)

where \( \lambda \) is the positive constant. The main target in this methodology is to keep the sliding surface slope \( s(x, t) \) near to the zero. Therefore, one of the common strategies is to find input \( U \) outside of \( s(x, t) \).

\[ \frac{1}{2} \frac{d}{dt} s^2(x, t) \leq -\zeta |s(x, t)| \]  

(6)

where \( \zeta \) is positive constant.

If \( S(0)>0 \rightarrow \frac{d}{dt} S(t) \leq -\zeta \)  

(7)

To eliminate the derivative term, it is used an integral term from \( t=0 \) to \( t=t_{reach} \)

\[ \int_{t=0}^{t=t_{reach}} \frac{d}{dt} S(t) \leq -\int_{t=0}^{t=t_{reach}} \eta \rightarrow S(t_{reach}) - S(0) \leq -\zeta (t_{reach} - 0) \]  

(8)

Where \( t_{reach} \) is the time that trajectories reach to the sliding surface so, suppose \( S(t_{reach} = 0) \) defined as

\[ 0 - S(0) \leq -\eta(t_{reach}) \rightarrow t_{reach} \leq \frac{S(0)}{\zeta} \]  

(9)

and

\[ if \ S(0) < 0 \rightarrow 0 - S(0) \leq -\eta(t_{reach}) \rightarrow S(0) \leq -\zeta(t_{reach}) \rightarrow t_{reach} \leq \frac{|S(0)|}{\eta} \]  

(10)

Equation (10) guarantees time to reach the sliding surface is smaller than \( \frac{|S(0)|}{\zeta} \) since the trajectories are outside of \( S(t) \).

\[ if \ S_{t_{reach}} = S(0) \rightarrow error(x - x_d) = 0 \]  

(11)

suppose \( S \) is defined as

\[ s(x, t) = (\frac{d}{dt} + \lambda) \ddot{x} = (\ddot{x} - \ddot{x}_d) + \lambda(x - x_d) \]  

(12)

The derivation of \( S \), namely, \( \dot{S} \) can be calculated as the following;

\[ \dot{S} = (\ddot{x} - \ddot{x}_d) + \lambda(\dot{x} - \dot{x}_d) \]  

(13)

suppose the second order system is defined as;

\[ \ddot{x} = f + u \rightarrow \ddot{\hat{x}} = f + \hat{U} - \ddot{x}_d + \lambda(\dot{x} - \dot{x}_d) \]  

(14)

Where \( f \) is the dynamic uncertain, and also since \( \hat{S} = 0 \) and \( \dot{\hat{S}} = 0 \), to have the best approximation \( \hat{U} \) is defined as

\[ \hat{U} = -\hat{f} + \ddot{x}_d - \lambda(\dot{x} - \dot{x}_d) \]  

(15)

A simple solution to get the sliding condition when the dynamic parameters have uncertainty is the switching control law:

\[ U_{dis} = \hat{U} - K(\ddot{x}, t) \cdot sgn(s) \]  

(16)
where the switching function \( \text{sgn}(S) \) is defined as
\[
\text{sgn}(s) = \begin{cases} 
  1 & s > 0 \\
  -1 & s < 0 \\
  0 & s = 0 
\end{cases}
\] (17)
and the \( K(x, t) \) is the positive constant. Suppose by (12) the following equation can be written as,
\[
\frac{1}{2} \frac{d}{dt} s^2(x, t) = S \cdot S = \left[ f - \hat{f} - K \text{sgn}(s) \right] \cdot S = \left( f - \hat{f} \right) \cdot S - K|S| 
\] (18)
and if the equation (9) instead of (8) the sliding surface can be calculated as
\[
s(x, t) = \left( \frac{d}{dt} + \lambda \right)^2 \left( \int_0^t \ddot{x} \, dt \right) = (\ddot{x} - \ddot{x}_d) + 2\lambda(x - x_d) - \lambda^2(x - x_d)
\] (19)
in this method the approximation of \( \ddot{U} \) is computed as
\[
\ddot{U} = -\dddot{f} + \ddot{x}_d - 2\lambda(x - x_d) + \lambda^2(x - x_d)
\] (20)
To reduce or eliminate the chattering it is used the boundary layer method; in boundary layer method the basic idea is replace the discontinuous method by saturation (linear) method with small neighborhood of the switching surface.
\[
B(t) = \{ x, |S(t)| \leq \phi \}; \phi > 0
\] (21)
Where \( \phi \) is the boundary layer thickness. Therefore the saturation function \( \text{Sat}(S/\phi) \) is added to the control law as
\[
U = K(x, t) \cdot \text{Sat}(S/\phi)
\] (22)
Where \( \text{Sat}(S/\phi) \) can be defined as [12-14]
\[
\text{Sat}(S/\phi) = \begin{cases} 
  1 & (S/\phi > 1) \\
  -1 & (S/\phi < 1) \\
  S/\phi & (-1 < S/\phi < 1)
\end{cases}
\] (23)
Based on above discussion, the control law for a multi degrees of freedom robot manipulator is written as:
\[
U = U_{eq} + U_r
\] (24)
Where, the model-based component \( \ddot{r}_{eq} \) is compensated the nominal dynamics of systems. Therefore \( \ddot{r}_{eq} \) can calculate as follows:
\[
U_{eq} = M^{-1}(P_m(\theta) + P_{net}(\theta)) + S|M
\] (25)
Where
\[
M^{-1} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}
\]
Figure 1 is shown the chattering free sliding mode controller applied in IC engine.
However SMC has satisfactory performance in certain or limited uncertain system but calculate the sliding surface slope in uncertain unstructured system by try and error or experience knowledge is very difficult; sliding mode fuzzy gain scheduling sliding mode controller is recommended.

\[ U_{SP} = \psi \left( K \cdot (mx + b) \cdot \left( \frac{S}{\gamma} \right) \right) \]  (26)

Where \( U_{SP} \) is sliding mode fuzzy output function. The adaption low is defined as

\[ \theta_j = \gamma_{sj} S \xi_j(S_j) \]  (27)

where the \( \gamma_{sj} \) is the positive constant and \( \xi_j(S_j) = [\xi_j^1(S_j), \xi_j^2(S_j), \xi_j^3(S_j), \ldots, \xi_j^M(S_j)]^T \)

\[ \xi_j^i(S_j) = \frac{\mu_{(A)j}(S_j)}{\sum_i \mu_{(A)j}(S_j)} \]  (28)

The fuzzy inference approximator can be defined as below

\[ f(x) = U_{fuzzy} = \sum_{i=1}^{M} \theta_i^T \xi(x) = \psi(S) \]  (29)

where \( \theta = (\theta^1, \theta^2, \theta^3, \ldots, \theta^M)^T, \xi(x) = (\xi^1(x), \xi^2(x), \xi^3(x), \ldots, \xi^M(x))^T \)

\[ \xi^i(x) = \frac{\sum_i \mu_{(A)j}(x)}{\sum_i \mu_{(A)j}(x)} \]  (30)

where \( \theta = (\theta^1, \theta^2, \theta^3, \ldots, \theta^M) \) is adjustable parameter and \( \mu_{(x)} \) is membership function.

error base fuzzy controller can be defined as

\[ U_{fuzzy} = \psi(S) \]  (31)

The fuzzy division can be reached the best state when \( S, \dot{S} < 0 \) and the error is minimum by the following formulation

\[ \theta^* = \arg \min_{\theta \in \Theta} \left[ \sup_{x \in U} \left| \sum_{i=1}^{M} \theta_i^T \xi(x) - U_{eg} \right|\right] \]  (32)

Where \( \theta^* \) is the minimum error, \( \sup_{x \in U} \left| \sum_{i=1}^{M} \theta_i^T \xi(x) - \tau_{equ} \right| \) is the minimum approximation error. suppose \( K_f \) is defined as follows
\[ K_j = \frac{\sum_{l=1}^{M} \theta_l^j \mu_{A_j}(S_j)}{\sum_{l=1}^{M} \mu_{A_j}(S_j)} = \theta_j^j \zeta_j(S_j) \]  

(33)

Where \( \zeta_j(S_j) = [\zeta_j^1(S_j), \zeta_j^2(S_j), \zeta_j^3(S_j), \ldots, \zeta_j^M(S_j)]^T \)

\[ \zeta_j^i(S_j) = \frac{\mu_{A_j}^i(S_j)}{\sum_{l=1}^{M} \mu_{A_j}^l(S_j)} \]  

(34)

where the \( \gamma_j \) is the positive constant.

According to the nonlinear dynamic equivalent formulation of robot manipulator the nonlinear equivalent part is estimated by (29)

\[ [M^{-1}(P_m(\theta) + P_{net}(\theta)) + \dot{S}]M = \sum_{l=1}^{M} \theta^T \zeta(x) - \lambda S - K \]  

(35)

Based on (27) the formulation of proposed fuzzy sliding mode controller can be written as [28];

\[ U = U_{eq_fuzzy} + U_r \]  

(36)

Where \( U_{eq_fuzzy} = [M^{-1}(P_m(\theta) + P_{net}(\theta)) + \dot{S}]M + \sum_{l=1}^{M} \theta^T \zeta(x) + K \)

As a result AFSGSMC is very stable with a good performance. Figure 2 is shown the block diagram of proposed AFSGSMC.

Fig. 2. Proposed adaptive fuzzy sliding gain scheduling SMC algorithm: applied to IC engine

To validate this work it is used IC engine and implements proposed AFSGSMC and SMC in this IC engine. The simulation was implemented in Matlab/Simulink environment. Fuel ratio trajectory and disturbance rejection are compared in these controllers.
**Fuel ratio trajectory:** Figure 3 is shown the fuel ratio in proposed AFSGSMC and SMC in uncertain environment for desired step input.

By comparing this response, Figure 3, in SMC and AFSGSMC, in certain environment both of controllers have about the same response. The Settling time in AFSGSMC is fairly lower than SMC.

**Disturbance rejection:** It is noted that, these systems are tested by band limited white noise with a predefined 40% of relative to the input signal amplitude. This type of noise is used to external disturbance in continuous and hybrid systems. Figure 4 is indicated the power disturbance removal in SMC and AFSGSMC. Besides a band limited white noise with predefined of 40% the power of input signal is applied to the trajectory response SMC and AFSGSMC; it found slight oscillations in classical SMC trajectory responses.

Among above graph, relating to desired trajectory following with structure and unstructured disturbance, SMC has slightly fluctuations.

5. **Conclusion**

Refer to the research, a position on-line fuzzy sliding gain scheduling sliding mode control (AFSGSMC) design and application to internal combustion engine has proposed in order to design high performance nonlinear controller in the presence of uncertainties and external disturbance. Regarding to the positive points in sliding mode algorithm and gain scheduling methodology which applied to sliding mode methodology and adaptive fuzzy sliding gain scheduling sliding mode control, the response is improved. In supervisory controller fuzzy logic method by adding to the sliding mode controller has covered negative points. Obviously IC engine is nonlinear and MIMO system so in proposed controller in first step design model based controller based on sliding mode controller and after that disturbance rejection is improved by adaptive fuzzy sliding mode gain scheduling sliding mode controller. Higher implementation quality of response and model based controller versus an acceptable performance in chattering and trajectory is reached by designing proposed adaptive controller and applied to IC engine. As a result, this controller will be able to control a wide range of IC engine with a high sampling rates because its easy to implement.

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