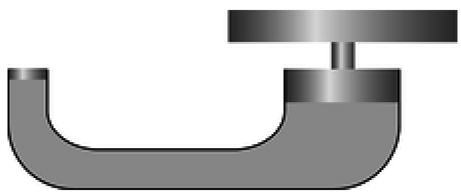
EQUILIBRIUM AND STABILITY

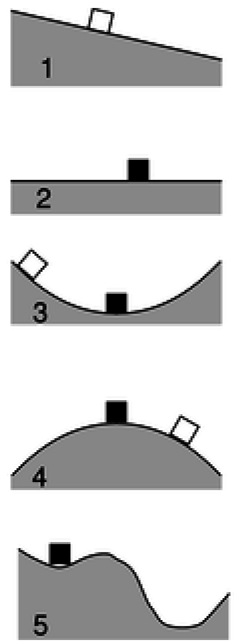
The seesaw in figure k is in equilibrium, meaning that if it starts out being at rest, it will stay put. This is known as a neutral equilibrium, since the seesaw has no preferred position to which it will return if we disturb it. If we move it to a different position and release it, it will stay at rest there as well. If we put it in motion, it will simply continue in motion until one person's feet hit the ground.



n / A hydraulic jack.

Most objects around you are in stable equilibria, like the black block in figure o/3. Even if the block is moved or set in motion, it will oscillate about the equilibrium position. The pictures are like graphs of y versus x, but since the gravitational energy $U=mgy$ is proportional to y, we can just as well think of them as graphs of U versus x. The block's stable equilibrium position is where the function $U(x)$ has a local minimum. The book you're reading right now is in equilibrium, but gravitational energy isn't the only form of energy involved. To move it upward, we'd have to supply gravitational energy, but downward motion would require a different kind of energy, in order to compress the table more.

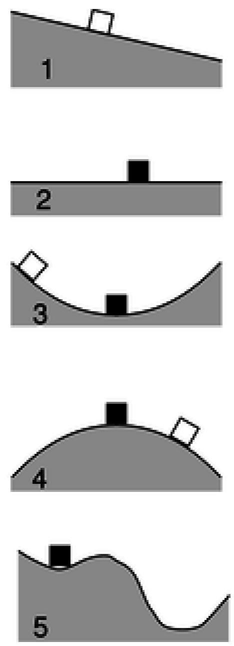
(As we'll see in section 2.4, this is electrical energy due to interactions between atoms within the table.)



o / The surfaces are frictionless. The black blocks are in equilibrium.

A differentiable function's local extrema occur where its derivative is zero. A position where dU/dx is zero can be a stable (3), neutral (2), or unstable equilibrium, (4). An unstable equilibrium is like a pencil balanced on its tip. Although it could theoretically remain balanced there forever, in reality it will topple due to any tiny perturbation, such as an air current or a vibration from a passing truck. This is a technical, mathematical definition of instability, which is more restrictive than the way the word is used in ordinary speech.

Most people would describe a domino standing upright as being unstable, but in technical usage it would be considered stable, because a certain finite amount of energy is required to tip it over, and perturbations smaller than that would only cause it to oscillate around its equilibrium position.



o / The surfaces are frictionless. The black blocks are in equilibrium.

The domino is also an interesting example because it has two local minima, one in which it is upright, and another in which it is lying flat. A local minimum that is not the global minimum, as in figure o/5, is referred to as a metastable equilibrium.

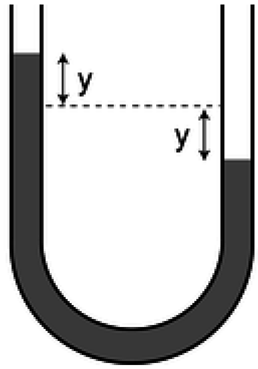
A neutral equilibrium

p / Example 12.

Figure p shows a special-purpose one-block funicular railroad near Hill and Fourth Streets in Los Angeles, California, used for getting passengers up and down a very steep hill. It has two cars attached to a single loop of cable, arranged so that while one car goes up, the other comes down. They pass each other in the middle.

Since one car's gravitational energy is increasing while the other's is decreasing, the system is in neutral equilibrium. If there were no frictional heating, exactly zero energy would be required in order to operate the system. A similar counterweighting principle is used in aerial tramways in mountain resorts, and in elevators (with a solid weight, rather than a second car, as counterweight).

Water in a U-shaped tube



q / Water in a U-shaped tube.

▹ The U-shaped tube in figure q has cross-sectional area A, and the density of the water inside is ρ. Find the gravitational energy as a function of the quantity y shown in the figure, and show that there is an equilibrium at y=0.

▹ The question is a little ambiguous, since gravitational energy is only well defined up to an additive constant. To fix this constant, let's define U to be zero when y=0. The difference between U(y) and U(0) is the energy that would be required to lift a water column of height y out of the right side, and place it above the dashed line, on the left side, raising it through a height y. This water column has height y and cross-sectional area A, so its volume is Ay, its mass is ρAy, and the energy required is mgy=(ρAy)gy=ρgAy2. We then have U(y)=U(0)+ρgAy2=ρgAy2.

To find equilibria, we look for places where the derivative dU/dy=2ρgAy equals 0. As we'd expect intuitively, the only equilibrium occurs at y=0. The second derivative test shows that this is a local minimum (not a maximum or a point of inflection), so this is a stable equilibrium.

Source:
http://physwiki.ucdavis.edu/Fundamentals/02._Conservation_of_Energy/2.1_Energy