

# Electrostatic Potential

## 11.1.1 Define electric potential

If an electric charge is moved in an electric field work is done on the charge. There is an electric force and the your displacing the charge...

To arrive at an equation for the electric potential we do the exactly as we did for gravitational potential, i.e. we need to employ a little calculus. If we move a positive charge  $q_1$  from infinity towards a stationary positive charge  $q_2$  to the point  $R$ , we are applying a force on  $q_1$ , but the displacement is in the opposite direction, so we need a negative sign...

$$W = F \cdot r \quad (1)$$

$$W = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \times -dr \quad (2)$$

$$W = - \int_{\infty}^R \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} dr = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} \Big|_{\infty}^R \quad (3)$$

$$W = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} \quad (4)$$

gives us an expression for the work done. From this we can say that two charges  $q_1$  and  $q_2$  when separated by a distance  $r$  have the potential energy equal to:

$$E_p = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} \quad (5)$$

The electric potential is defined as the potential energy per unit charge:

$$V = \frac{E_p}{q} = k \frac{q}{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad (6)$$

The last equation gives us an expression for the work done. From this we can say that two charges  $q_1$  and  $q_2$  when separated by a distance  $r$  have the potential energy equal to:

$$E_p = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} \quad (7)$$

The electric potential is defined as the potential energy per unit charge:

$$V = \frac{E_p}{q} = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad (8)$$

The units of electric potential is Volts, this is the same unit we talk about in terms of electric circuits. Potential is always measured between two points in an electric field or relative to something. When we say the power in the sockets is 220 volts, we mean relative to ground... We can only measure relative difference in potential, we can not measure absolute potential.

It is important to note:

1. Electric potential, like gravitational potential, is a scalar.
2. The potential from multiple sources adds.

### ***11.1.2 Determine the electric potential due to various charge configurations.***

The electric potential due to a point charge can be found using the equation:

(9)

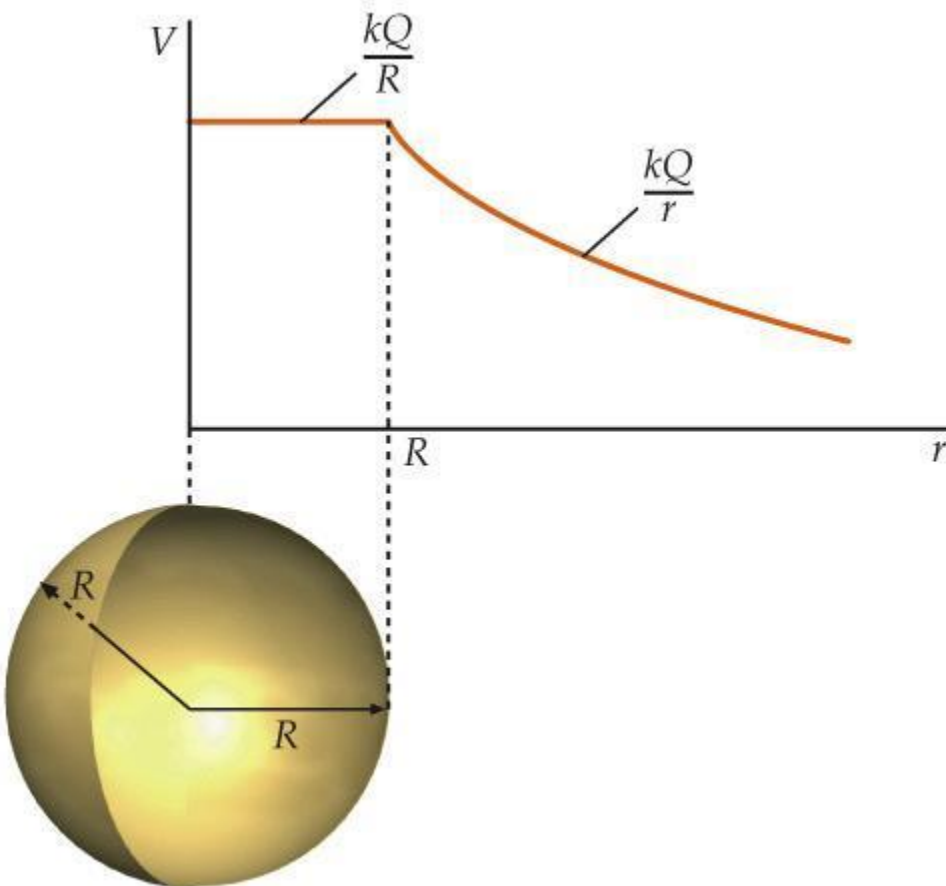
$$V = q / 4\pi\epsilon_0 r$$

If there is more than one point charge the electric potential due to the different charges adds simply due to the scalar nature of electric potential. For example given two charges  $q_1$  and  $q_2$  the electric potential at a point  $r_1$  and  $r_2$  from the respective charges can be found by:

$$V = \frac{q_1}{4\pi\epsilon_0 r} + \frac{q_2}{4\pi\epsilon_0 r} \quad (10)$$

The electric potential outside of a charged sphere is the same as for a point charge. Outside the sphere the sphere appears (electrically) to be a point charge at the center of the sphere.

Technically you don't need to know this: Inside a conducting sphere the potential is constant...



Stolen from: [http://www3.ltu.edu/~s\\_schneider/courses/univ2/images/figure-23-15.gif](http://www3.ltu.edu/~s_schneider/courses/univ2/images/figure-23-15.gif)

### ***11.1.3 State and apply the formula relating electric field strength to potential gradient***

If a charge is in an electric field and moved a small distance then work is done on that charge. The force done is:

(11)

$$W = Eq\Delta x$$

Where  $q$  is the charge and  $Eq$  is the force on the particle. The work done on the particle is equal to the energy added or the change in the energy of the particle:

(12)

$$\Delta \text{Energy} = Eq \Delta x$$

Above we defined the energy of a charged particle in an electric field to be the voltage times the charge therefore:

(13)

$$\Delta Vq = Eq \Delta x$$

On the left there is only a change in the voltage, the charge will not change as the particle is moved in the through the electric field. If we solve the equation for  $E$  the charges cancel and we get the following:

(14)

$$E = \Delta V / \Delta x$$

This gives us a direct relationship between the voltage and the electric field. It is often easier to calculate or measure the voltage than to calculate or measure the electric field. This formula allows us to derive the electric field knowing the voltage difference between two points separated by a distance .

#### ***11.1.4 Describe the similarities and differences between gravitational fields and electrical fields***

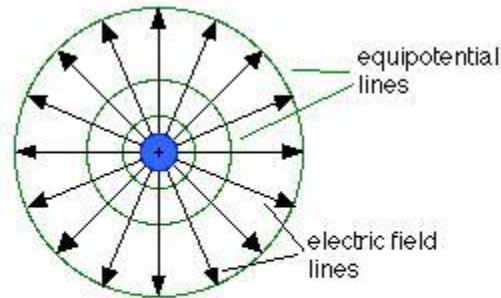
I really don't want to. Here's a website summarizing the similarities and differences (link is broken, but maybe they'll fix it?).

[http://www.s-cool.co.uk/topic\\_quicklearn.asp?loc=ql&topic\\_id=14&quicklearn\\_id=5&subject\\_id=2&ebt=181&ebn=&ebs=&ebl=&elc=13](http://www.s-cool.co.uk/topic_quicklearn.asp?loc=ql&topic_id=14&quicklearn_id=5&subject_id=2&ebt=181&ebn=&ebs=&ebl=&elc=13)

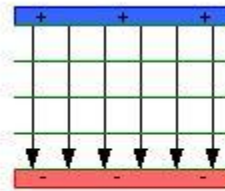
#### ***11.1.5 Describe and sketch patterns of equipotential surfaces***

#### ***11.1.6 Explain the relation of equipotential surfaces to electric field lines.***

Points in space that have a constant value for the electric potential are called equipotential surfaces. Fascinating. We can draw lines of equipotential, the lines of equipotential are always perpendicular to the electric field lines.

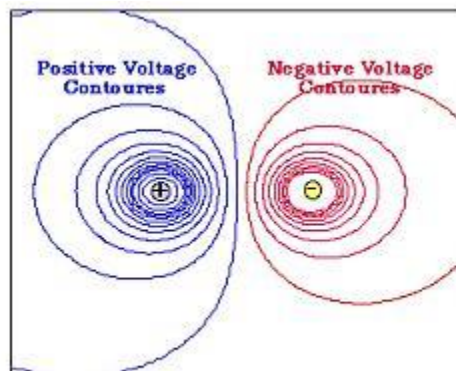


Field and equipotential lines for a positive point charge

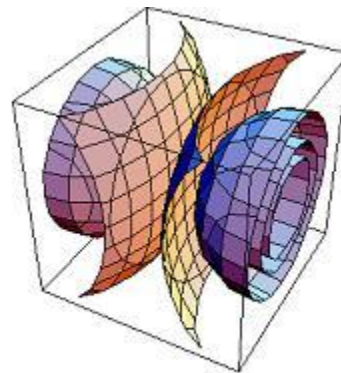


Field and equipotential lines for a set of parallel plates

Stolen from: <http://web.uccs.edu/physics112/Chapter%2025.htm>



Equipotential surface from opposite charges in 2-D.



Equipotential surfaces from two opposite charges in 3-D

Source: <http://ibphysicsstuff.wikidot.com/electrostatic-potential>