

# Module 6

## Magnetic Circuits and Core Losses

# Lesson 22

## Eddy Current & Hysteresis Loss

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# Chapter 22

## Eddy Current & Hysteresis Losses (Lesson 22)

### 22.1 Lesson goals

In this lesson we shall show that (i) a time varying field will cause *eddy currents* to be induced in the core causing power loss and (ii) *hysteresis* effect of the material also causes additional power loss called *hysteresis loss*. The effect of both the losses will make the core hotter. We must see that these two losses, (together called *core loss*) are kept to a minimum in order to increase efficiency of the apparatus such as transformers & rotating machines, where the core of the magnetic circuit is subjected to time varying field. If we want to minimize something we must know the origin and factors on which that something depends. In the following sections we first discuss *eddy current phenomenon* and then the phenomenon of *hysteresis*.

Finally expressions for (i) inductance, (ii) stored energy density in a magnetic field and (iii) force between parallel faces across the air gap of a magnetic circuit are derived.

*Key Words:* Hysteresis loss; hysteresis loop; eddy current loss; Faraday's laws;

After going through this section students will be able to answer the following questions. After going through this lesson, students are expected to have clear ideas of the following:

1. Reasons for core losses.
2. That core loss is sum of hysteresis and eddy current losses.
3. Factors on which hysteresis loss depends.
4. Factors on which eddy current loss depends.
5. Effects of these losses on the performance of magnetic circuit.
6. How to reduce these losses?
7. Energy storing capability in a magnetic circuit.
8. Force acting between the parallel faces of iron separated by air gap.
9. Iron cored inductance and the factors on which its value depends.

### 22.2 Introduction

While discussing magnetic circuit in the previous lesson (no. 21) we assumed the exciting current to be constant d.c. We also came to know how to calculate flux ( $\phi$ ) or flux density ( $B$ ) in the core for a constant exciting current. When the exciting current is a function of time, it is expected that flux ( $\phi$ ) or flux density ( $B$ ) will be functions of time too, since  $\phi$  produced depends on  $i$ . In addition if the current is also alternating in nature then both the

magnitude of the flux and its direction will change in time. The magnetic material is now therefore subjected to a time varying field instead of steady constant field with d.c excitation. Let:

$$\begin{aligned} \text{The exciting current } i(t) &= I_{max} \sin \omega t \\ \text{Assuming linearity, flux density } B(t) &= \mu_0 \mu_r H(t) \\ &= \mu_0 \mu_r \frac{Ni}{l} \\ &= \mu_0 \mu_r \frac{N I_{max} \sin \omega t}{l} \\ \therefore B(t) &= B_{max} \sin \omega t \end{aligned}$$

### 22.2.1 Voltage induced in a stationary coil placed in a time varying field

If normal to the area of a coil, a time varying field  $\phi(t)$  exists as in figure 22.1, then an emf is induced in the coil. This emf will appear across the free ends 1 & 2 of the coil. Whenever we talk about some voltage or emf, two things are important, namely the magnitude of the voltage and its polarity. Faraday's law tells us about the both. Mathematically it is written as  $e(t) = -N \frac{d\phi}{dt}$

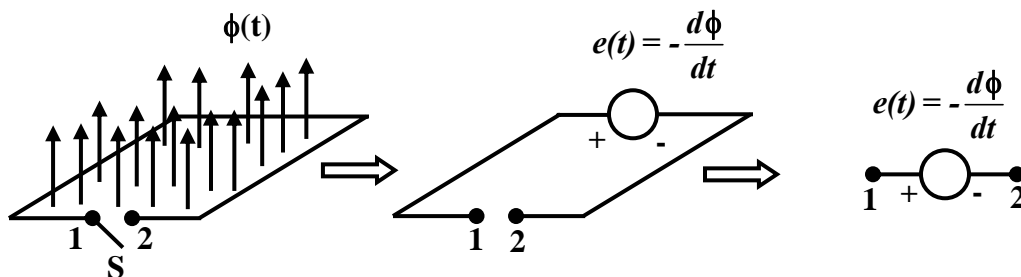


Figure 22.1:

Let us try to understand the implication of this equation a bit deeply.  $\phi(t)$  is to be taken normal to the surface of the coil. But a surface has two *normals*; one in the upward direction and the other in downward direction for the coil shown in the figure. Which one to take? The choice is entirely ours. In this case we have chosen the normal along the upward direction. This direction is obtained if you start your journey from the terminal-2 and reach the terminal-1 in the anticlockwise direction along the contour of the coil. Once the direction of the normal is chosen what we have to do is to express  $\phi(t)$  along the same direction. Then calculate  $N \frac{d\phi}{dt}$  and put a - ve sign before it. The result obtained will give you  $e_{12}$  i.e., potential of terminal-1 wrt terminal-2. In other words, the whole coil can be considered to be a source of emf wrt terminals 1 & 2 with polarity as indicated. If at any time flux is increasing with time in the upward direction,  $\frac{d\phi}{dt}$  is + ve and  $e_{12}$  will come out to be - ve as well at that time. On the other hand, at any time flux is decreasing with time in the upward direction,  $\frac{d\phi}{dt}$  is - ve and  $e_{12}$  will come out to be + ve as well at that time. Mathematically let:

$$\begin{aligned}
 \text{Flux density } B(t) &= B_{max} \sin \omega t \\
 \text{Area of the coil} &= A \\
 \text{Flux crossing the area } \phi(t) &= B(t) A \\
 &= B_{max} A \sin \omega t \\
 &= \phi_{max} \sin \omega t \\
 \text{Induced voltage in the coil } e_{12} &= -N \frac{d\phi}{dt} \\
 &= -1 \times \frac{d\phi}{dt} \quad \because N=1 \text{ here} \\
 &= \phi_{max} \omega \cos \omega t \\
 \therefore e_{12} &= E_{max} \cos \omega t \\
 \text{RMS value of } e_{12} \text{ } E &= \frac{\phi_{max} \omega}{\sqrt{2}} \\
 \therefore E &= \sqrt{2} \pi f \phi_{max} \text{ putting } \omega = 2\pi f
 \end{aligned}$$

If the switch S is closed, this voltage will drive a circulating current  $i_c$  in the coil the direction of which will be such so as to oppose the cause for which it is due. Correct instantaneous polarity of the induced voltage and the direction of the current in the coil are shown in figure 22.2, for different time intervals with the switch S closed. In the interval  $0 < \omega t < \frac{\pi}{2}$ ,  $\frac{d\phi}{dt}$  is +ve

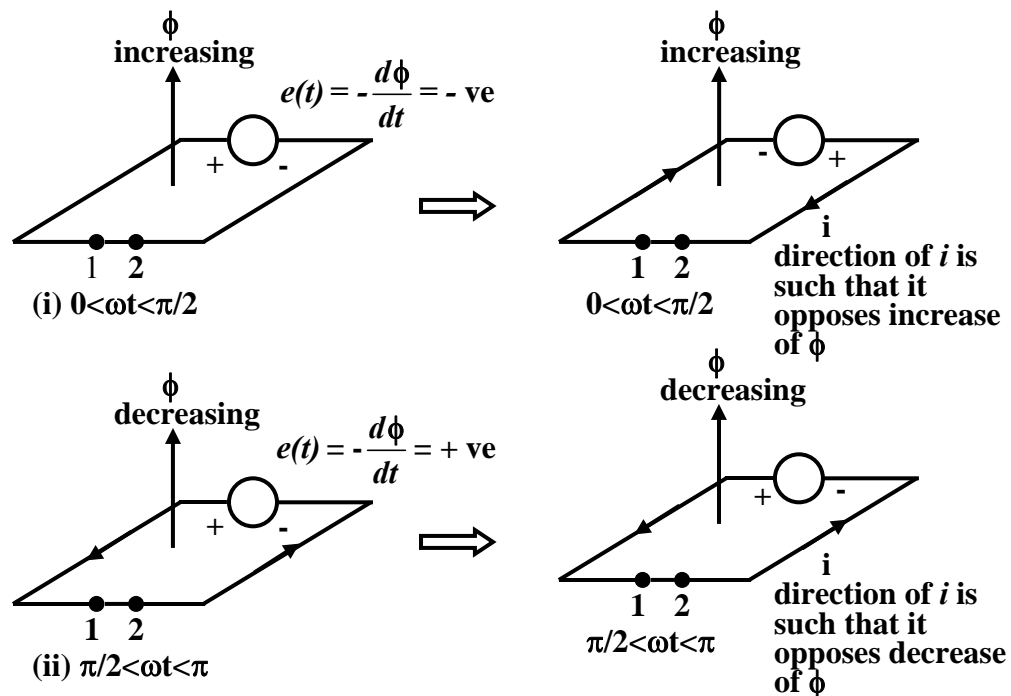


Figure 22.2: Direction of induced current.

### 22.2.2 Eddy current

Look at the Figure 22.3 where a rectangular core of magnetic material is shown along with the exciting coil wrapped around it. Without any loss of generality, one may consider this to be a part of a magnetic circuit. If the coil is excited from a sinusoidal source, exciting current flowing will be sinusoidal too. Now put your attention to any of the cross section of the core and imagine any arbitrary rectangular closed path  $abcd$ . An emf will be induced in the path  $abcd$  following Faraday's law. Here of course we don't require a switch  $S$  to close the path because the path is closed by itself by the conducting magnetic material (say iron). Therefore a circulating current  $i_{eddy}$  will result. The direction of  $i_{eddy}$  is shown at the instant when  $B(t)$  is increasing with time. It is important to note here that to calculate induced voltage in the path, the value of flux to be taken is the flux enclosed by the path i.e.,  $\phi_{max} = B_{max} \times \text{area of the loop } abcd$ . The magnitude of the eddy current will be limited by the *path resistance*,  $R_{path}$  neglecting reactance effect. Eddy current will therefore cause power loss in  $R_{path}$  and heating of the core. To calculate the total eddy current loss in the material we have to add all the power losses of different eddy paths covering the whole cross section.

### 22.2.3 Use of thin plates or laminations for core

We must see that the power loss due to eddy current is minimized so that heating of the core is reduced and efficiency of the machine or the apparatus is increased. It is obvious if the cross sectional area of the eddy path is reduced then eddy voltage induced too will be reduced ( $E_{eddy} \propto \text{area}$ ), hence eddy loss will be less. This can be achieved by using several thin electrically insulated plates (called laminations) stacked together to form the core instead a solid block of iron. The idea is depicted in the Figure 22.4 where the plates have been shown for clarity, rather separated from each other. While assembling the core the *laminations* are kept closely pact. Conclusion is that solid block of iron should not be

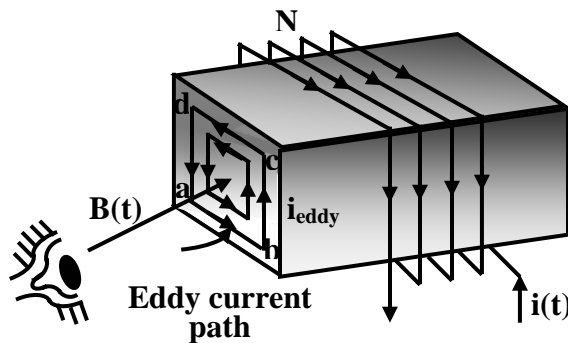


Figure 22.3: Eddy current paths

used to construct the core when exciting current will be ac. However, if exciting current is dc, the core need not be laminated.

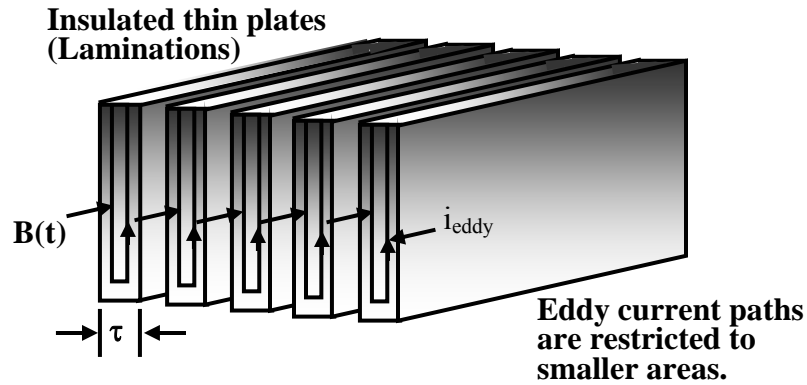


Figure 22.4: Laminated core to reduce eddy loss.

### 22.2.4 Derivation of an expression for eddy current loss in a thin plate

From physical consideration we have seen that *thin* plates each of thickness  $\tau$ , are to be used to reduce eddy loss. With this in mind we shall try to derive an approximate expression for eddy loss in the following section for a thin plate and try to identify the factors on which it will depend. Section of a thin plate  $\tau \ll L$  and  $h$  is shown in the plane of the screen in Figure 22.5.

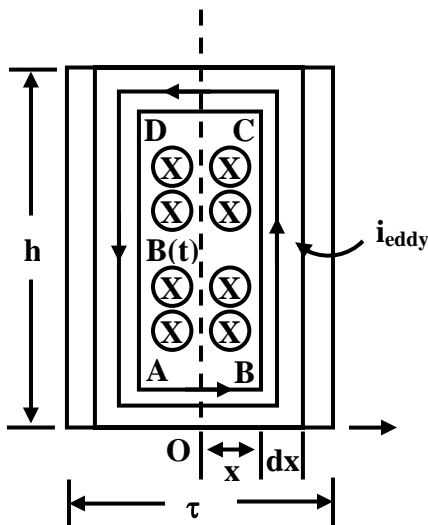


Figure 22.5: Elemental eddy current path.

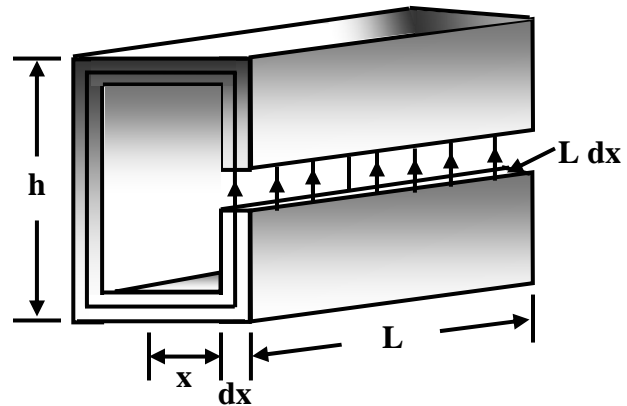


Figure 22.6: Section of the elemental eddy current path.

Eddy current loss is essentially  $I^2R$  loss occurring inside the core. The current is caused by the induced voltage in any conceivable closed path due to the time varying field as shown in the diagram 22.5.

Let us consider a thin magnetic plate of length  $L$ , height  $h$  and thickness  $\tau$  such that  $\tau$  is very small compared to both  $L$  and  $h$ . Also let us assume a sinusoidally time varying field  $b = B_{\max} \sin \omega t$  exists perpendicular to the rectangular area formed by  $\tau$  and  $h$  as shown in figure 22.5.

Let us consider a small elemental rectangular closed path ABCDA of thickness  $dx$  and at a distance  $x$  from the origin. The loop may be considered to be a single coil through which time varying flux is crossing. So there will be induced voltage in it, in



similar manner as voltage is induced in a coil of single turn shown in the previous section. Now,

$$\text{Area of the loop ABCD} = 2hx$$

$$\text{Flux crossing the loop} = B_{max} 2hx \sin \omega t$$

$$\text{RMS voltage induced in the loop, } E = \sqrt{2} \pi f B_{max} 2hx$$

$$\text{Resistance of the path through which eddy current flows, } R_{path} = \frac{\rho(2h + 4x)}{L dx}$$

To derive an expression for the eddy current loss in the plate, we shall first calculate the power loss in the elemental strip and then integrate suitably to find total loss. Power loss in the loop  $dP$  is given by:

$$\begin{aligned} dP &= \frac{E^2}{R_{path}} \\ &= \frac{E^2 L dx}{\rho(2h + 4x)} \\ &= \frac{E^2 L dx}{\rho 2h} \text{ since } \tau \ll h \end{aligned}$$

$$\begin{aligned} \text{Total eddy current loss, } P_{eddy} &= \frac{4\pi^2 B_{max}^2 f^2 hL}{\rho} \int_{x=0}^{\frac{\tau}{2}} x^2 dx \\ &= \frac{\pi^2 f^2 B_{max}^2 \tau^2}{6\rho} (hL\tau) \end{aligned}$$

$$\text{Volume of the thin plate} = hL\tau$$

$$\text{Eddy loss per unit volume, } P_{eddy} = \frac{\pi^2 f^2 B_{max}^2 \tau^2}{6\rho}$$

$$\text{or, } P_{eddy} = k_e f^2 B_{max}^2 \tau^2$$

Thus we find eddy current loss per unit volume of the material directly depends upon the *square* of the *frequency*, *flux density* and *thickness* of the plate. Also it is inversely proportional to the *resistivity* of the material. The core of the material is constructed using thin plates called laminations. Each plate is given a *varnish coating* for providing necessary insulation between the plates. *Cold Rolled Grain Oriented*, in short CRGO sheets are used to make transformer core.

## 22.3 Hysteresis Loss

### 22.3.1 Unidirectional time varying exciting current

Consider a magnetic circuit with constant (d.c) excitation current  $I_0$ . Flux established will have fixed value with a fixed direction. Suppose this final current  $I_0$  has been attained from zero current slowly by energizing the coil from a potential divider arrangement as depicted in Figure 22.7. Let us also assume that initially the core was not magnetized. The exciting current therefore becomes a function of time till it reached the desired current  $I$  and we stopped further increasing it. The flux too naturally will be function of time and cause induced voltage  $e_{12}$  in the coil with a polarity to oppose the increase of inflow of current as shown. The coil becomes a source of emf with terminal-1, +ve and terminal-2, -ve. Recall that a source in which current enters through its +ve terminal absorbs power or energy while it delivers power or energy when current comes out of the +ve terminal. Therefore during the interval when  $i(t)$  is increasing the coil absorbs energy. Is it possible to know how much energy does the coil absorb when current is increased from 0 to  $I_0$ ? This is possible if we have the B-H curve of the material with us.

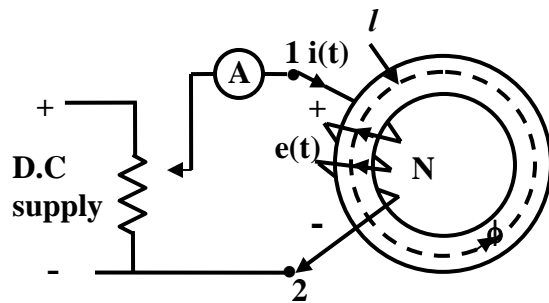


Figure 22.7:

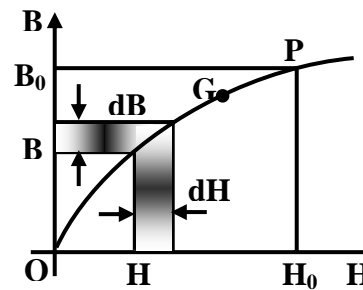


Figure 22.8:

### 22.3.2 Energy stored, energy returned & energy density

Let:

$$i = \text{current at time } t$$

$$H = \text{field intensity corresponding to } i \text{ at time } t$$

$$B = \text{flux density corresponding to } i \text{ at time } t$$

(22.1)

Let an infinitely small time  $dt$  elapses so that new values become:

$$i + di = \text{Current at time } t + dt$$

$$H + dH = \text{Field intensity corresponding to } i + di \text{ at time } t + dt$$

$$B + dB = \text{Flux density corresponding to } i + di \text{ at time } t + dt$$

$$\text{Voltage induced in the coil } e_{12} = N \frac{d\phi}{dt}$$

$$\begin{aligned}
 &= NA \frac{dB}{dt} \\
 \text{Power absorbed at } t &= e_{12}i \\
 &= NA \frac{dB}{dt} i \\
 &= Al H \frac{dB}{dt} \text{ noting, } H = \frac{Ni}{l} \\
 \text{Energy absorbed in time } dt, dW &= Al H \frac{dB}{dt} i \times dt \\
 &= Al H dB \\
 \text{total energy absorbed per unit} &= \int_0^{B_0} H dB \\
 \text{volume, } W &
 \end{aligned}$$

Graphically therefore, the closed area  $OKPB_0BO$  is a measure of the energy stored by the field in the core when current is increased from 0 to  $I_0$ . What happens if now current is gradually reduced back to 0 from  $I_0$ ? The operating point on B-H curve does not trace back the same path when current was increasing from 0 to  $I_0$ . In fact, B-H curve (PHT) remains above during decreasing current with respect the B-H curve (OGP) during increasing current as shown in figure 22.9. This lack of retracing the same path of the curve is called *hysteresis*. The portion OGP should be used for increasing current while the portion (PHT) should be used for decreasing current. When the current is brought back to zero external applied field  $H$  becomes zero and the material is left magnetized with a residual field  $OT$ . Now the question is when the exciting current is decreasing, does the coil absorb or return the energy back to supply. In this case  $\frac{dB}{dt}$  being  $-ve$ , the induced voltage reverses its polarity although direction of  $i$  remains same. In other words, current leaves from the  $+ve$  terminal of the induced voltage thereby returning power back to the supply. Proceeding in the same fashion as adopted for increasing current, it can be shown that the area  $PMTRP$  represents amount of energy returned per unit volume. Obviously energy absorbed during rising current from 0 to  $I_0$  is more than the energy returned during lowering of current from  $I_0$  to 0. The balance of the energy then must have been lost as heat in the core.

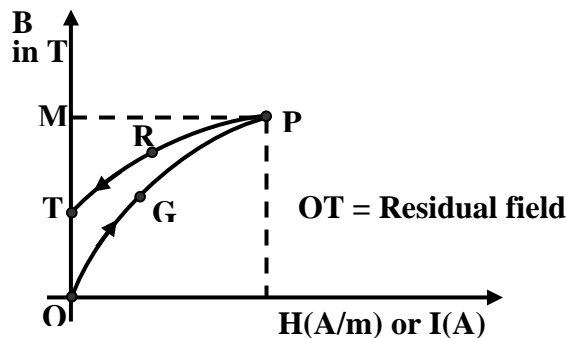


Figure 22.9:

## 22.4 Hysteresis loop with alternating exciting current

In the light of the above discussion, let us see how the operating point is traced out if the exciting current is  $i = I_{max} \sin \omega t$ . The nature of the current variation in a complete cycle can be enumerated as follows:

In the interval  $0 \leq \omega t \leq \frac{\pi}{2}$  :  $i$  is +ve and  $\frac{di}{dt}$  is +ve.

In the interval  $\frac{\pi}{2} \leq \omega t \leq \pi$  :  $i$  is +ve and  $\frac{di}{dt}$  is -ve.

In the interval  $\pi \leq \omega t \leq \frac{3\pi}{2}$  :  $i$  is -ve and  $\frac{di}{dt}$  is -ve.

In the interval  $\frac{3\pi}{2} \leq \omega t \leq 2\pi$  :  $i$  is -ve and  $\frac{di}{dt}$  is +ve.

Let the core had no residual field when the coil is excited by  $i = I_{max} \sin \omega t$ . In the interval  $0 < \omega t < \frac{\pi}{2}$ , B will rise along the path OGP. Operating point at P corresponds to  $+I_{max}$  or  $+H_{max}$ . For the interval  $\frac{\pi}{2} < \omega t < \pi$  operating moves along the path PRT. At point T, current is zero. However, due to sinusoidal current,  $i$  starts increasing in the -ve direction as shown in the Figure 22.10 and operating point moves along TSEQ. It may be noted that a -ve H of value OS is necessary to bring the residual field to zero at S. OS is called the *coercivity* of the material. At the end of the interval  $\pi < \omega t < \frac{3\pi}{2}$ , current reaches  $-I_{max}$  or field  $-H_{max}$ . In the next interval,  $\frac{3\pi}{2} < \omega t < 2\pi$ , current changes from  $-I_{max}$  to zero and operating point moves from M to N along the path MN. After this a new cycle of current variation begins and the operating point now never enters into the path OGP. The movement of the operating point can be described by two paths namely: (i) QFMNKP for increasing current from  $-I_{max}$  to  $+I_{max}$  and (ii) from  $+I_{max}$  to  $-I_{max}$  along PRTSEQ.

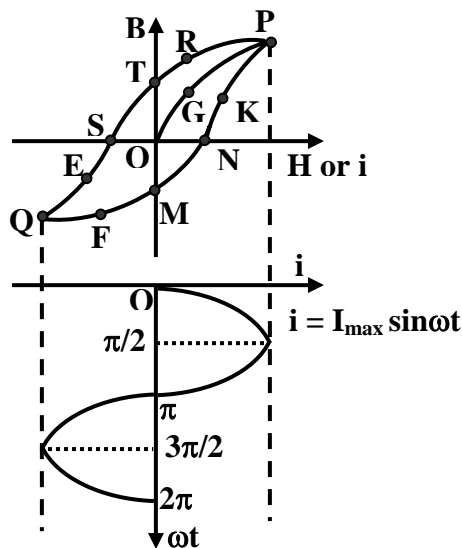


Figure 22.10: B-H loop with sinusoidal current.

### 22.4.1 Hysteresis loss & loop area

In other words the operating point trace the *perimeter* of the closed area QFMNKPRTSEQ. This area is called the B-H loop of the material. We will now show that the area enclosed by the loop is the *hysteresis loss* per unit volume per cycle variation of the current. In the interval  $0 \leq \omega t \leq \frac{\pi}{2}$ ,  $i$  is +ve and  $\frac{di}{dt}$  is also +ve, moving the operating point from M to P along the path MNKP. Energy *absorbed* during this interval is given by the shaded area MNKPLTM shown in Figure 22.11 (i).

In the interval  $\frac{\pi}{2} \leq \omega t \leq \pi$ ,  $i$  is +ve but  $\frac{di}{dt}$  is -ve, moving the operating point from P to T along the path PRT. Energy *returned* during this interval is given by the shaded area PLTRP shown in Figure 22.11 (ii). Thus during the +ve half cycle of current variation net amount of energy absorbed is given by the shaded area MNKPRTM which is nothing but half the area of the loop.

In the interval  $\pi \leq \omega t \leq \frac{3\pi}{2}$ ,  $i$  is -ve and  $\frac{di}{dt}$  is also -ve, moving the operating point from T to Q along the path TSEQ. Energy *absorbed* during this interval is given by the shaded area QJMTSEQ shown in Figure 22.11 (iii).

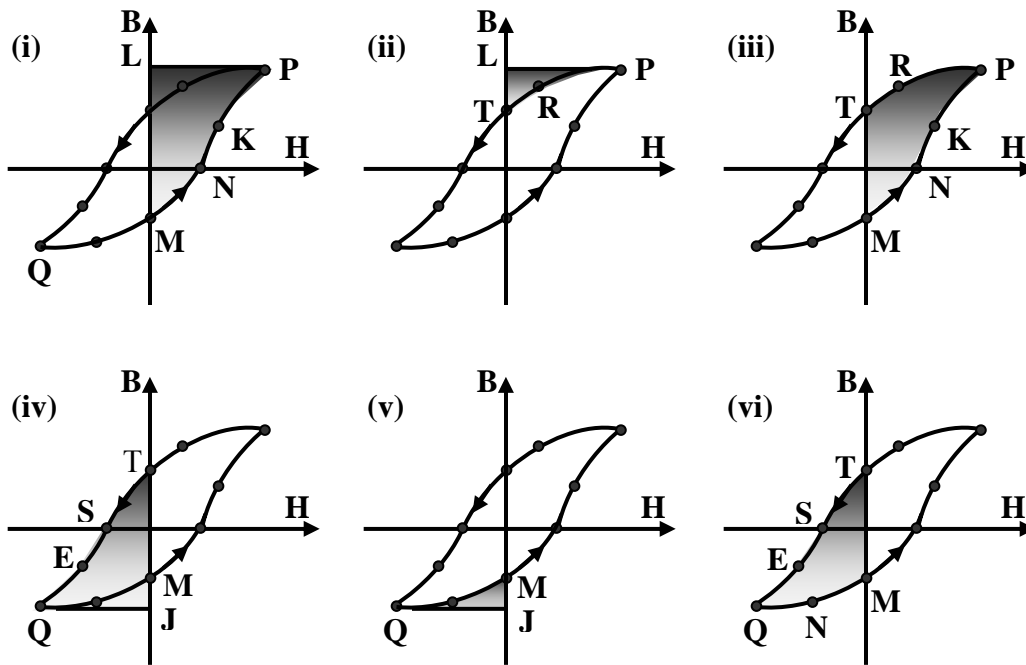


Figure 22.11: B-H loop with sinusoidal current.

In the interval  $\frac{3\pi}{2} \leq \omega t \leq 2\pi$ ,  $i$  is -ve but  $\frac{di}{dt}$  is +ve, moving the operating point from Q to M along the path QEM. Energy *returned* during this interval is given by the shaded area QJMFQ shown in Figure 22.11 (iv).

Thus during the -ve half cycle of current variation net amount of energy absorbed is given by the shaded area QFMTSEQ which is nothing but the other half the loop area.

Therefore total area enclosed by the B-H loop is the measure of the hysteresis loss per unit volume per unit cycle. To reduce hysteresis loss one has to use a core material for which area enclosed will be as small as possible.

## Steinmetz's empirical formula for hysteresis loss

Based on results obtained by experiments with different ferromagnetic materials with sinusoidal currents, Charles Steinmetz *proposed* the empirical formula for calculating hysteresis loss analytically.

$$\text{Hysteresis loss per unit volume, } P_h = k_h f B_{\max}^n$$

Where, the coefficient  $k_h$  depends on the material and  $n$ , known as Steinmetz exponent, may vary from 1.5 to 2.5. For iron it may be taken as 1.6.

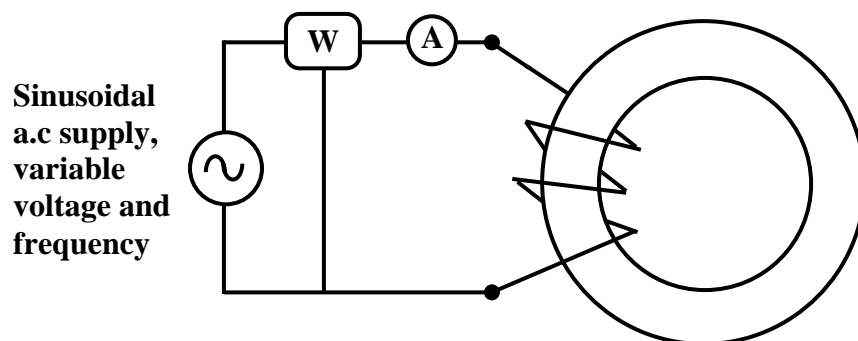
## 22.5 Separation of core loss

The sum of hysteresis and eddy current losses is called core loss as both the losses occur within the core (magnetic material). For a given magnetic circuit with a core of ferromagnetic material, volume and thickness of the plates are constant and the total core loss can be expressed as follows.

Core loss = Hysteresis loss + Eddy current loss

$$P_{\text{core}} = K_h f B_{\max}^n + K_e f^2 B_{\max}^2$$

It is rather easier to measure the core loss with the help of a wattmeter (W) by energizing the  $N$  turn coil from a sinusoidal voltage of known frequency as shown in figure 22.12.



**Figure 22.12: Core loss measurement.**

Let  $A$  be the cross sectional area of the core and let winding resistance of the coil be negligibly small (which is usually the case), then equating the applied rms voltage to the induced rms voltage of the coil we get:

$$V \approx \sqrt{2}\pi f \phi_{\max} N$$

$$\text{Or, } V = \sqrt{2}\pi f B_{\max} AN$$

$$\text{So, } B_{\max} = \frac{V}{\sqrt{2}\pi f AN}$$

$$\therefore B_{\max} \propto \frac{V}{f}$$

The above result i.e.,  $B_{\max} \propto \frac{V}{f}$  is important because it tells us that to keep  $B_{\max}$  constant at rated value at lower frequency of operation, applied voltage should be proportionately decreased. In fact, from the knowledge of  $N$  (number of turns of the coil) and  $A$  (cross sectional area of the core),  $V$  (supply voltage) and  $f$  (supply frequency) one can estimate the maximum value of the flux density from the relation  $B_{\max} = \frac{V}{\sqrt{2}\pi f AN}$ . This point has been further discussed in the future lesson on transformers.

Now coming back to the problem of separation of core loss into its components: we note that there are three unknowns, namely  $K_h$ ,  $K_e$  and  $n$  (Steinmetz's exponent) to be determined in the equation  $P_{\text{core}} = K_h f B_{\max}^n + K_e f^2 B_{\max}^2$ . LHS of this equation is nothing but the wattmeter reading of the experimental set up shown in Figure 22.12. Therefore, by noting down the wattmeter readings corresponding to three different applied voltages and frequencies, we can have three independent algebraic equations to solve for  $K_h$ ,  $K_e$  and  $n$ . However, to simplify the steps in solving of the equations two readings may be taken at same flux density (keeping  $\frac{V}{f}$  ratio constant) and the third one at different flux density. To understand this, solve the following problem and verify the answers given.

For a magnetic circuit, following results are obtained.

Frequency	$B_{\max}$	Core loss
50 Hz	1.2 T	115 W
30 Hz	1.2 T	60.36 W
30 Hz	1.4 T	87.24 W

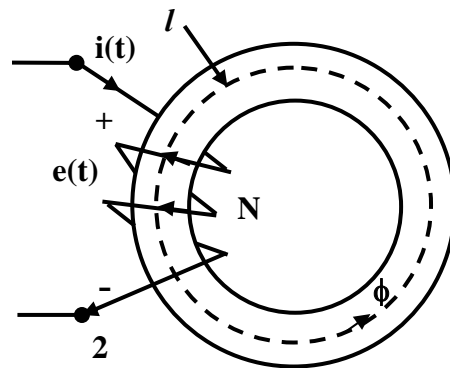
Estimate the constants,  $K_h$ ,  $K_e$  and  $n$  and separate the core loss into hysteresis and eddy losses at the above frequencies and flux densities.

The answer of the problem is:

Frequency	$B_{\max}$	Core loss	Hyst loss	Eddy loss
50 Hz	1.2 T	115 W	79 W	36 W
30 Hz	1.2 T	60.36 W	47.4 W	12.96 W
30 Hz	1.4 T	87.24 W	69.6 W	17.64 W

## 22.6 Inductor

One can make an inductor  $L$ , by having several turns  $N$ , wound over a core as shown in figure 22.13. In an *ideal* inductor, as we all know, no power loss takes place. Therefore, we must use a very good magnetic material having negligible B-H loop area. Also we must see that the operating point lies in the linear zone of the B-H characteristic in order to get a constant value of the inductance. This means  $\mu_r$  may be assumed to be constant. To make eddy current loss vanishingly small, let us assume the lamination thickness is extremely small and the core material has a very high resistivity  $\rho$ . Under these assumptions let us derive an expression for the inductance  $L$ , in order to have a feeling on the factors it will depend upon. Let us recall that inductance of a coil is defined as the flux linkage with the coil when 1 A flows through it.



**Figure 22.13: An inductor.**

Let  $\phi$  be the flux produced when  $i$  A flows through the coil. Then by definition:

$$\begin{aligned}
 \text{Total flux linkage} &= N\phi \\
 \therefore \text{inductance is } L &= \frac{N\phi}{i} \text{ by definition.} \\
 &= \frac{N B A}{i} \quad \because \phi = B \times A \\
 &= \frac{N \mu_0 \mu_r H A}{i} \quad \because B = \mu_0 \mu_r H \\
 &= \mu_0 \mu_r \frac{N H A}{i} \\
 &= \mu_0 \mu_r \frac{N \frac{Ni}{l} A}{i} \quad \text{putting } H = \frac{Ni}{l} \\
 \text{Finally, } L &= \mu_0 \mu_r \frac{N^2 A}{l}
 \end{aligned}$$

The above equation relates *inductance* with the dimensions of the magnetic circuit, number of turns and permeability of the core in the similar way as we relate



resistance of a wire, with the dimensions of the wire and the resistivity (recall,  $R = \rho \frac{l}{A}$ ). It is important to note that  $L$  is directly proportional to the square of the number of turns, directly proportional to the sectional area of the core, directly proportional to the permeability of the core and inversely proportional to the mean length of the flux path. In absence of any core loss and linearity of B-H characteristic, Energy stored during increasing current from 0 to  $I$  is exactly equal to the energy returned during decreasing current from  $I$  to 0. From our earlier studies we know for increasing current:

$$\begin{aligned}
 \text{Voltage induced in the coil } e &= N A \frac{dB}{dt} \\
 \text{Energy absorbed in time } dt \text{ is } dW &= e i dt = N A \frac{dB}{dt} i dt \\
 &= N A \frac{dB}{dt} i dt \\
 &= N i A dB \\
 &= A l H dB \\
 \text{Energy absorbed to reach } I \text{ or } B &= A l \int_0^B H dB \\
 &= A l \int_0^B \frac{B}{\mu_0 \mu_r} dB \\
 &= A l \frac{B^2}{2 \mu_0 \mu_r} \\
 \text{Energy stored per unit volume} &= \frac{B^2}{2 \mu_0 \mu_r}
 \end{aligned}$$

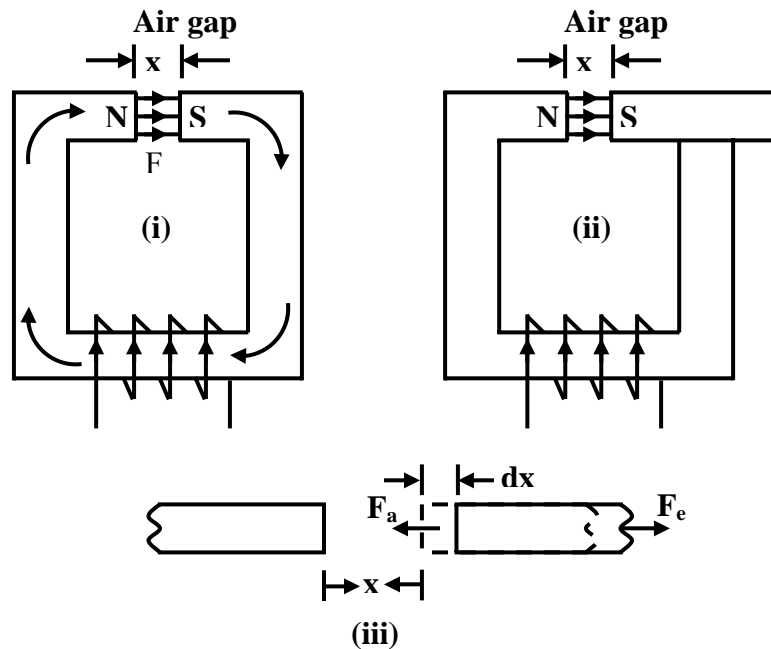
By expressing B in terms of current,  $I$  in the above equation one can get a more familiar expression for energy stored in an inductor as follows:

$$\begin{aligned}
 \text{Energy absorbed to reach } I \text{ or } B &= A l \frac{B^2}{2 \mu_0 \mu_r} \\
 &= A l \frac{(\mu_0 \mu_r H)^2}{2 \mu_0 \mu_r} \\
 &= A l \frac{\mu_0 \mu_r H^2}{2} \\
 &= A l \mu_0 \mu_r \frac{(N I)^2}{2 l^2} \\
 &= \frac{1}{2} \left( \frac{\mu_0 \mu_r A N^2}{l} \right) I^2 \\
 \therefore \text{Energy stored in the inductor} &= \frac{1}{2} L I^2
 \end{aligned}$$

## 22.7 Force between two opposite faces of the core across an air gap

In a magnetic circuit involving *air gap*, magnetic force will exist between the parallel faces across the air gap of length  $x$ . The situation is shown for a magnetic circuit in figure 22.14 (i). Direction of the lines of forces will be in the clockwise direction and the left face will become a north pole and the right face will become a south pole. Naturally there will be force of *attraction*  $F_a$  between the faces. Except for the fact that this force will develop stress in the core, no physical movement is possible as the structure is *rigid*.

$$\begin{aligned}
 \text{Let the flux density in the air gap be} &= B \\
 \text{energy stored per unit volume in the gap} &= \frac{B^2}{2\mu_0} \\
 \text{gap volume} &= A x \\
 \text{Total energy stored} &= \frac{B^2}{2\mu_0} \times \text{gap volume} \\
 &= \frac{B^2}{2\mu_0} A x
 \end{aligned}
 \tag{22.2}$$



**Figure 22.14: Force between parallel faces.**

Easiest way to derive expression for  $F_a$  is to apply law of conservation of energy by using the concept of *virtual work*. To do this, let us *imagine* that right face belongs to a freely moving structure with initial gap  $x$  as in figure 22.14 (ii). At this gap  $x$ , we have find  $F_a$ . Obviously if we want to displace the moving structure by an elemental distance  $dx$  to the right, we have apply a force  $F_e$  toward right. As  $dx$  is very small tending to 0, we can assume  $B$  to remain unchanged. The magnitude of this external force  $F_e$  has to be

same as the prevailing force of attraction  $F_a$  between the faces. Where does the energy expended by the external agency go? It will go to *increase* the energy stored in the gap as its volume increase by  $A dx$ . Figure 22.14 (iii) shows an expanded view of the gap portion for clarity. Let us put it in mathematical steps as follows:

$$\begin{aligned} \text{energy stored per unit volume in the gap} &= \frac{B^2}{2\mu_0} \\ \text{initial gap volume} &= A x \\ \text{Total energy stored, } W_x &= \frac{B^2}{2\mu_0} \times \text{gap volume} \\ W_x &= \frac{B^2}{2\mu_0} A x \\ \text{let the external force applied be} &= F_e \\ \text{let the force of attraction be} &= F_a \\ \text{as explained above, } F_e &= F_a \\ \text{work done by external agency} &= F_e dx = F_a dx \\ \text{increase in the volume of the gap} &= A (x + dx) - Ax = A dx \\ \text{increase in stored energy} &= \frac{B^2}{2\mu_0} A dx \\ \text{but work done by external agency} &= \text{increase in stored energy} \\ F_a dx &= \frac{B^2}{2\mu_0} A dx \\ \text{or, desired force of attraction } F_a &= \frac{B^2}{2\mu_0} A \end{aligned}$$

## 22.8 Tick the correct answer

- If the number of turns of a coil wound over a core is halved, the inductance of the coil will become:

(A) doubled. (B) halved. (C) quadrupled. (D)  $\frac{1}{4}$  th

- The expression for eddy current loss per unit volume in a thin ferromagnetic plate of thickness  $\tau$  is:

$$\begin{aligned} \text{(A)} \quad \frac{1}{6\rho} \pi^2 f^2 B_{\max}^2 \tau^2 & \quad \text{(B)} \quad \frac{\rho}{6} \pi^2 f^2 B_{\max}^2 \tau^2 \\ \text{(C)} \quad \frac{1}{6\rho} \pi^2 f^2 B_{\max} \tau^2 & \quad \text{(D)} \quad \frac{1}{6\rho} \pi^2 f B_{\max}^2 \tau^2 \end{aligned}$$

3. As suggested by Steinmetz, hysteresis loss in a ferromagnetic material is proportional to:

$$(A) f^n B_{\max} \quad (B) f B_{\max}^n$$

$$(C) f^2 B_{\max}^2 \quad (D) f B_{\max}^n$$

where,  $n$  may vary between 1.5 to 2.5 depending upon material.

4. The eddy current loss in a magnetic circuit is found to be 100 W when the exciting coil is energized by 200 V, 50 Hz source. If the coil is supplied with 180 V, 54 Hz instead, the *eddy current loss* will become

$$(A) 90 \text{ W} \quad (B) 81 \text{ W} \quad (C) 108 \text{ W} \quad (D) 50 \text{ W}$$

5. A magnetic circuit draws a certain amount of alternating sinusoidal exciting current producing a certain amount of alternating flux in the core. If an air gap is introduced in the core path, the exciting current will:

$$(A) \text{ increase.} \quad (B) \text{ remain same.} \quad (C) \text{ decrease.} \quad (D) \text{ vanish}$$

## 22.9 Solve the following

- The area of the hysteresis loop of a  $1200 \text{ cm}^3$  ferromagnetic material is  $0.9 \text{ cm}^2$  with  $B_{\max} = 1.5 \text{ T}$ . The scale factors are  $1 \text{ cm} \equiv 10 \text{ A/m}$  along x-axis and  $1 \text{ cm} = 0.8 \text{ T}$  along y-axis. Find the power loss in watts due to hysteresis if this material is subjected to an 50 Hz alternating flux density with a peak value 1.5 T.
- Calculate the core loss per kg in a specimen of alloy steel for a maximum density of 1.1 T and a frequency of 50 Hz, using 0.4 mm plates. Resistivity  $\rho$  is  $24 \mu \Omega\text{-cm}$ ; density is  $7.75 \text{ g/cm}^3$ ; hysteresis loss  $355 \text{ J/m}^3$  per cycle.
- (a) A linear magnetic circuit has a mean flux length of 100 cm and uniform cross sectional area of  $25 \text{ cm}^2$ . A coil of 100 turns is wound over it and carries a current of 0.5 A. If relative permeability of the core is 1000, calculate the inductance of the coil and energy stored in the coil.  
  
(b) In the magnetic circuit of part (a), if an air gap of 2 mm length is introduced calculate (i) the energy stored in the air gap (ii) energy stored in core and (iii) force acting between the faces of the core across the gap for the same coil current.
- An iron ring with a mean diameter of 35 cm and a cross section of  $17.5 \text{ cm}^2$  has 110 turns of wire of negligible resistance. (a) What voltage must be applied to the coil at 50 Hz to obtain a maximum flux density of 1.2 T; the excitation required corresponding to this density 450 AT/m? Find also the inductance. (b) What is the effect of introducing a 2 mm air gap?

5. A coil wound over a core, is designed for 200 V (rms), 50 Hz such that the operating point is on the knee of the B-H characteristic of the core. At this rated voltage and frequency the value of the exciting current is found to be 1 A. Give your comments on the existing current if the coil is energized from:
- (a) 100 V, 25 Hz supply.
  - (b) 200 V, 25 Hz supply.

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