DIFFERENTIAL EQUATIONS OF PHYSICAL SYSTEMS

The term mechanical translation is used to describe motion with a single degree of freedom or motion in a straight line. The basis for all translational motion analysis is Newton's second law of motion which states that the Net force F acting on a body is related to its mass M and acceleration 'a' by the equation $\Sigma F = Ma$

'Ma' is called reactive force and it acts in a direction opposite to that of acceleration. The summation of the forces must of course be algebraic and thus considerable care must be taken in writing the equation so that proper signs prefix the forces.

The three basic elements used in linear mechanical translational systems are (i) Masses (ii) springs iii) dashpot or viscous friction units. The graphical and symbolic notations for all three are shown in fig 1-8

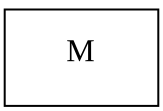


Fig 1-8 a) Mass

Fig 1-8 b) Spring

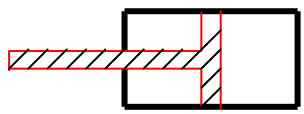
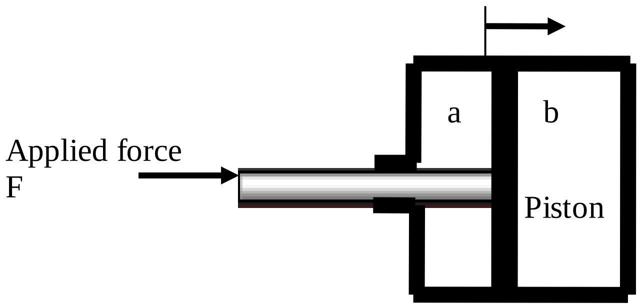


Fig 1-8 c) Dashpot

The spring provides a restoring a force when a force F is applied to deform a coiled spring a reaction force is produced, which to bring it back to its freelength. As long as deformation is small, the spring behaves as a linear element. The reaction force is equal to the product of the stiffness k and the amount of deformation.

Whenever there is motion or tendency of motion between two elements, frictional forces exist. The frictional forces encountered in physical systems are usually of nonlinear nature. The characteristics of the frictional forces between two contacting surfaces often depend on the composition of the surfaces. The pressure between surfaces, their relative velocity and others. The friction encountered in physical systems may be of many types

(coulomb friction, static friction, viscous friction) but in control problems viscous friction, predominates. Viscous friction represents a retarding force i.e. it acts in a direction opposite to the velocity and it is linear relationship between applied force and velocity. The mathematical expression of viscous friction $F=BV$ where B is viscous frictional co-efficient. It should be realized that friction is not always undesirable in physical systems. Sometimes it may be necessary to introduce friction intentionally to improve dynamic response of the system. Friction may be introduced intentionally in a system by use of dashpot as shown in fig 1-9. In automobiles shock absorber is nothing but dashpot.



The basic operation of a dashpot, in which the housing is filled with oil. If a force f is applied to the shaft, the piston presses against oil increasing the pressure on side ‘b’ and decreasing pressure side ‘a’ As a result the oil flows from side ‘b’ to side ‘a’ through the wall clearance. The friction coefficient B depends on the dimensions and the type of oil used.

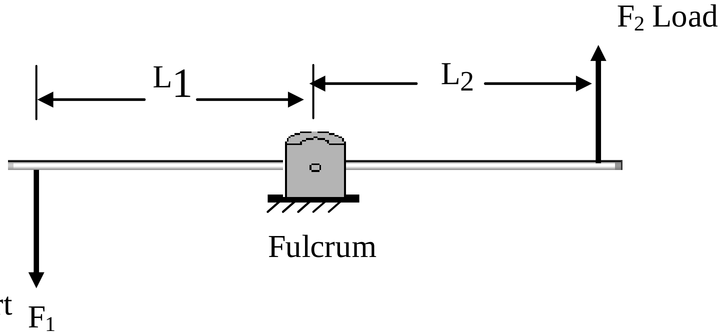
Outline of the procedure

For writing differential equations

1. Assume that the system originally is in equilibrium in this way the often-troublesome effect of gravity is eliminated.
2. Assume then that the system is given some arbitrary displacement if no distributing force is present.
3. Draw a freebody diagram of the forces exerted on each mass in the system. There should be a separate diagram for each mass.
4. Apply Newton’s law of motion to each diagram using the convention that any force acting in the direction of the assumed displacement is positive is positive.
5. Rearrange the equation in suitable form to solve by any convenient mathematical means.

Lever

Lever is a device which consists of rigid bar which tends to rotate about a fixed point called ‘fulcrum’ the two arms are called “effort arm” and “Load arm” respectively. The lever bears analogy with transformer



It is also called ‘mechanical transformer’

Equating the moments of the force

$F_1 L_1 = F_2 L_2$

$$F_2 = \frac{F_1 L_1}{L_2}$$