Contribution of Maxwell Stress in Air on the Deformations of Induction Machines

K. A. Fonteyn†, A. Belahcen*, P. Rasilo*, R. Kouhia** and A. Arkkio*

Abstract – Deformations in a cage-induction machine are investigated with simulations. The contribution of the Maxwell stress in the air gap and coil regions of the machine on the deformation is studied by comparing results obtained with and without inclusion of the stress into the calculation. The work attests the acceptability of an energy-based magneto-mechanical model for a 2D mesh of two different rotating electrical machines.

Keywords: Induction machine, Magnetoelasticity, Magnetostriction, Maxwell stress, Vibrations

1. Introduction

Nowadays, the computational power of computers has increased considerably making it possible to solve systems of equations with several thousands of variables such as required in the finite element method and especially, when coupling the magnetic and mechanical fields. Such a coupling is necessary when studying the behavior of the magnetic and mechanical field within electrical steels sheets composing electrical machines. Electrical steel sheets are mostly ferromagnetic materials. Ferromagnetism is a property of materials such as iron that enables interaction with magnets to take place. Such materials have a crystal structure and are thus subdivided into grains with uniform crystal structure but with different magnetic orientations [1-3].

Rotating electrical machines are subjected to forces of various origins such as mechanical and magnetic ones i.e. [5]. The IEEE Standard Dictionary of Electrical and Electronics Terms, defines a force as “any physical cause that is capable of modifying the motion of a body” [4]. The cross section of radial-flux electrical machines contains several regions that are not filled with magnetic material, such as the air gap and the slots containing the windings. Magnetic forces are classified in three groups. The first one contains the predominant forces that act on the boundary regions from the air onto the iron. These known as the reluctance forces or Maxwell forces. The second group gathers the forces having their source in the microscopic magnetic properties of the ferromagnetic material. The final third group gathers the forces, called the Lorentz forces. Those act on currents in the magnetic field.

This paper presents a method to take into account the stresses acting on the material boundaries, especially between air and iron, in magneto-mechanical calculations. It studies their contribution on the deformations of two different types of induction machines.

Modeling magneto-sensitive elastic solids like elastomers, and piezo-electric materials has been a subject of interest for many years. Some years ago, an interesting approach has been provided in [6] and [7]. A Helmholtz free energy, that accounts for the dependence of the magnetic field on the stress tensor and vice versa is introduced. The dependence of parameters within the constitutive equations is physically described. For this model, the use of an appropriately chosen set of invariants is proposed in [8] to represent the required behavior of the material.

2. Methods

Following partly the methodology described in [6] and [7] and focusing on a suitable choice of invariants, a Helmholtz free energy $\Psi$ is chosen so that the magneto-mechanical constitutive equations of the material are

$$\sigma = \rho \frac{\partial \Psi}{\partial \varepsilon}$$

$$M = -\rho \frac{\partial \Psi}{\partial B}$$

where $M$ is the magnetization vector and $\sigma$ is the Cauchy stress tensor, $\varepsilon$ is the strain tensor and $B$ is the magnetic flux density vector [1]. The derivation for this energy and justification for the model are described in [9-13] and for this reason are not recalled here. However, six parameters (denoted $\alpha_0 ... \alpha_5$) will arise from the derivation of the model and need to be identified such as discussed in [12].

With $I$ being the identity tensor, and $\mu_0$ the permeability in vacuum, the Maxwell stress tensor in magnetized matter

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is defined as:

$$\mathbf{\tau}_{\text{iron}} = \mu_0 \left( B \otimes B - \frac{1}{2} (\mathbf{B} \cdot \mathbf{B}) I \right) + (\mathbf{M} \cdot \mathbf{B}) \mathbf{I} - B \otimes \mathbf{M}$$  \hspace{1cm} (3)$$

The magnetization vector is trivially

$$\mathbf{M} = \mu_0 \mathbf{B} - \mathbf{H}$$  \hspace{1cm} (4)$$

In air, \( \mathbf{M} \) is null, so that, denoting \( \mathbf{\tau}_{\text{air}} \) as the Maxwell stress tensor in air, (3) becomes

$$\mathbf{\tau}_{\text{air}} = \mu_0 \left( B \otimes B - \frac{1}{2} (\mathbf{B} \cdot \mathbf{B}) I \right)$$  \hspace{1cm} (5)$$

To account for the penetration of the magnetic flux density in the air-gap, the electromagnetic stress tensor (5) is added to the total stress tensor in the total stiffness matrix for those elements having a common air-iron boundary. The total stress tensor evaluated for the nodes is

$$\mathbf{\tau} = \mathbf{\sigma} + \mathbf{\tau}_{\text{iron}} + \mathbf{\tau}_{\text{air}}$$  \hspace{1cm} (6)$$

Here, the numerical problem is solved in suitable finite element software [9].

The model requires parameter identification, and thus a clear knowledge of the material magneto-mechanical properties. For the simulated machines discussed later in Section 4, these data are not available. However, parameters from a typical steel sheet used in electrical machines are presented in the next section. The identified parameters will be used as a hypothesis, as an input for the finite element method.

### 3. Measurements and identification

The necessary parameters for the model have been identified from unidirectional magnetostrictive stress measurements from the modified Epstein frame. The device illustrated in Fig. 1 was modified from the IEC 60404-2 standard to apply a mechanical pre-stress to the iron strips [10, 12, 13]. The square-shaped frame comprises a primary winding and a secondary winding, the first one for feeding the coils, and the second one, to acquire the voltage to evaluate the magnetic flux density. Parameters for the setup and sample sheets are presented in Table 1. Four load cells, connected by means of screws on the strips to a measuring device that acquires the applied force are attached as in Fig. 1. The other four corners are fixed. Mechanical pre-stresses are applied and at equilibrium, the strips are magnetically excited and the resulting force is acquired with a piezo-electric sensor directly connected to the data acquisition card. Measured magnetostrictive curves at different values of compressive and tensile pre-stresses are presented in Fig. 2. The parameters of the described model in Section 2 have been identified from the set of curves and their values are gathered in Table 2.

<table>
<thead>
<tr>
<th>Table 1. Parameters for modified Epstein frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter Value Unit</td>
</tr>
<tr>
<td>Thickness of the sample 0.5 mm</td>
</tr>
<tr>
<td>Length of the sample 300 mm</td>
</tr>
<tr>
<td>Width of the sample 30 mm</td>
</tr>
<tr>
<td>Cross section area 225 mm$^2$</td>
</tr>
<tr>
<td>Resistance of primary coil 0.079 Ω</td>
</tr>
<tr>
<td>Inductance of primary coil 138 mH</td>
</tr>
<tr>
<td>Turns in primary coil 700</td>
</tr>
<tr>
<td>Resistance of secondary coil 1.347 Ω</td>
</tr>
<tr>
<td>Inductance of secondary coil 138 mH</td>
</tr>
<tr>
<td>Turns in secondary coil 700</td>
</tr>
</tbody>
</table>

![Fig. 1. Schematic view of the modified Epstein frame. 1-4: load cells, 5: piezo-electric sensor, 6: feeding coils and B coils, 7: test samples, 8: PC, 9: force meters, 10: air-flux compensating coil.](image)

![Fig. 2. Identified and measured magnetostrictive curves at different compressive and tensile pre-stresses.](image)
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Fig. 3. Simulated magnetic flux density distribution.

Fig. 4. Deformation of stator (left) when the electromagnetic stress tensor in the air gap is not taken into account, and (right) when it is taken into account. The flux path is shown to explain the existence of displacement ripple on positive side of displacements only (See Fig. 5)

4. Applications and results

The following simulations present an approximate idea of the deformation of a cage induction machine. In the simulations, first-order elements have been used to speed up the calculations as the fully coupled method requires more computation time than an uncoupled methodology. Machines I and II (Fig. 3) are standard cage induction motors and magnetic materials of the stator are fully processed non-oriented steel sheets. Data for Machines I and II are presented later in Table 3. The discussion on the solution will be based on relative values of displacements.

Magnetic flux-density distributions at an arbitrary time step are shown in Fig. 3. The corresponding original and displaced 2-D stator geometries are presented in Fig. 4. The differences between the last four figures are specifically discussed below.

In the next figures, six simulations have been performed. Each of them has taken a steady state of the machine into account. Indeed, to avoid any problem related to transient phenomena, an initial magnetic state for the time-stepping analysis was computed from the results of a time-harmonic formulation. For each simulation, the machines were supplied with a sinusoidal voltage at the rated values.

Table 3. Main parameters used as input for the simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Machine I</th>
<th>Machine II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated voltage</td>
<td>380 V</td>
<td>380 V</td>
</tr>
<tr>
<td>Slip</td>
<td>2 %</td>
<td>3.2 %</td>
</tr>
<tr>
<td>Rated current</td>
<td>60 A</td>
<td>27 A</td>
</tr>
<tr>
<td>Rated power</td>
<td>30 kW</td>
<td>15 kW</td>
</tr>
<tr>
<td>Supply frequency</td>
<td>50 Hz</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Number of pole pairs</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Number of phases</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Number of parallel paths</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Outer diameter of st core</td>
<td>323 mm</td>
<td>235 mm</td>
</tr>
<tr>
<td>Inner diameter of st core</td>
<td>190.2 mm</td>
<td>145 mm</td>
</tr>
<tr>
<td>Number of stator slots</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>Outer diameter of rt core</td>
<td>188.37 mm</td>
<td>144.1 mm</td>
</tr>
<tr>
<td>Number of rotor slots</td>
<td>28</td>
<td>34</td>
</tr>
</tbody>
</table>

Fig. 5. Machine I: Displacements in radial and tangential directions as function of time of a node on the tooth of the stator. Subscripts $r$ and $\theta$ stand for the radial and tangential directions. 1 refers to the case when only magnetostriction is considered. 2 is the case when both magnetostriction and the electromagnetic stress tensor in iron are used in the simulations.

In the first set of simulations, the influence of the elastic stress tensor and magnetostriction only on the displacements of the structure is analyzed. Then, the
simulations account additionally for the electromagnetic stress in iron such as in Figs. 5 and 7. Finally, the electromagnetic stress tensor in air is added to the calculation, hence the reluctance forces will be taken into account as they are dominant forces in electrical machines such as in Figs. 6 and 8.

The boundary conditions on the stator are such that the outer boundary of the machine is fixed in the tangential direction but free to move in the radial direction throughout the simulations. On the rotor, the boundary conditions are such that the shaft is fixed in both directions. To simplify the problem and because its influence is supposed to be minimum, the welding of the outer boundary of the machine is not taken into account in the simulations.

From the developed method, the displacements of each node of the mesh are known at every time step. From the results, the magnetostrictive effect seems attenuated by the contribution of the electromagnetic stress within the simulated electrical steel sheet, as in Figs. 5 and 7. Accordingly, the contribution of magnetostriction and magnetic stress in iron tends to expand the shape of the machine (left side of Fig. 4). When the electromagnetic stress tensor in iron is taken into account (or not) can be discussed thanks to the developed method. In general, the addition of electromagnetic stress to the computation reduces the peak-to-peak amplitudes of the displacements. It can be seen that the DC component is reduced by 20% for Machine I and by 25% for Machine II. The ripples in the displacements are more dominant for Machine II because of its geometry. Radial displacements in Machine II are higher than in Machine I because the magnetic flux density is higher. The effect of the electromagnetic stress is small with respect to the deformations resulting from the magnetostriction. Finally, the tangential displacements are three times smaller than the radial displacements on the tooth of the stator.

The addition of the electromagnetic stress tensor in air into the computation has an enormous influence on the displacements of the stator and the rotor. The above figures suggest that the displacements are increased up to 4 times, depending on the number of pole pairs and the magnitude of the magnetic flux density. The deformations resulting from the addition of this stress to the computation are shown in Figs. 5 and 7 for both machines.

The stress tensor, when used in iron only and not in air produces negative stresses on the surface of iron making it to shrink and the displacements thus to be smaller than in the case of magnetostriction only (See Fig. 5). When the contribution of the stress tensor from air is accounted for, the total stress on the iron surface is positive and produces forces that make the iron to stretch, which results in higher displacements (See Fig. 6). Further, the contribution of air
elements to the stress is much higher than that of iron, which results in higher displacements.

The ripple in the displacement of both machines is due to the rotor slot passing effect, which makes the flux distribution in the stator slot varying at higher frequency. This effect is stronger in Machine II due to the higher number of rotor slots and poles. In Figs. 5 and 6, the ripple shifted from the positive side of the displacement to the negative side when the stress contribution from air element is accounted for. This is due to phase-opposition between the displacements due to the magnetostriiction and these due to the stress tensor in air. Further the ripple is only on one side of the displacement because when the flux passes through a given tooth it also passes in the yoke causing positive displacement and it also contains higher harmonics, whereas the displacement is negative when the flux does not pass through the tooth and thus no harmonics are present. The flux path is illustrated in Fig. 4.

To complement this study, one node in the center of the stator tooth is presented in Figs. 6(b) and 8(b). When compared to the displacements on the outer boundary – Figs. 6(a) and 8(a) – it can be established that the displacements on the teeth in the radial direction are higher than those on the outer boundary of the stator. The displacements in the tangential directions are also relatively important. The results are plotted in Figs. 6(c) and 8(c).

5. Conclusion

According to the computed results two conclusions are established. First, based on the knowledge of the parameters of the material, the model is suitable for predicting the displacements in a radial-flux rotating electrical machine. Second, the displacements, when accounting for the Maxwell stress in air, are significantly high. In the study case the outer boundary of the stator undergoes a rather high displacement. The presented model has been identified with measurements on a modified Epstein frame and implemented into finite element software for the computation of electrical machines. The reluctance forces have been taken into account indirectly by dealing with the related stress tensor and their influence quantified. However such results require verifications by measurements on a real electrical machine. Such a work is the subject of a future paper as the presented model seems suitable for the computation of rotating electrical machines.

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