Capacitor Impedance

Shunt capacitors, either at the customer location for power factor correction or on the distribution system for voltage control, dramatically alter the system impedance variation with frequency. Capacitors do not create harmonics, but severe harmonic distortion can sometimes be attributed to their presence. While the reactance of inductive components increases proportionately to frequency, capacitive reactance $X_C$ decreases proportionately:

$$X_C = \frac{1}{\frac{2\pi f}{C}}$$

-------------- (4.9)

$C$ is the capacitance in farads. This quantity is seldom readily available for power capacitors, which are rated in terms of kvar or Mvar at a given voltage. The equivalent line-to-neutral capacitive reactance at fundamental frequency for a capacitor bank can be determined by

$$X_C = \frac{kV^2}{M\text{var}}$$

-------------- (4.10)

4.7.1 Parallel Resonance

All circuits containing both capacitances and inductances have one or more natural frequencies. When one of those frequencies lines up with a frequency that is being produced on the power system, a resonance may develop in which the voltage and current at that frequency continue to persist at very high values. This is the root of most problems with harmonic distortion on power systems.

Figure 4.23 shows a distribution system with potential parallel resonance problems. From the perspective of harmonic sources the shunt capacitor appears in parallel with the equivalent system inductance (source and transformer inductances) at harmonic frequencies as depicted in Fig. 4.24b.

Parallel resonance occurs when the reactance of $X_C$ and the distribution system cancel each other out. The frequency at which this phenomenon occurs is called the parallel resonant frequency. It can be expressed as follows:
\[ f_p = \frac{1}{2} \left( \frac{1}{R} - \frac{1}{L_{eq} C} \right) \left( \frac{R^2}{4L_{eq}^2} - \frac{1}{L_{eq} C} \right) \]  

------------------ (4.11)

Where \( R \) = resistance of combined equivalent source and transformer

\( L_{eq} \) = inductance of combined equivalent source and transformer

\( C \) = capacitance of capacitor bank

At the resonant frequency, the apparent impedance of the parallel combination of the equivalent inductance and capacitance as seen from the harmonic current source becomes very large, i.e.,

\[ Z = \frac{X_C}{X_{eq}} \left( \frac{X_{eq}}{X_{eq}} \right) \left( \frac{X_C}{X_{eq}} \right) \]

\[ X_C = \frac{X_{eq}}{X_{eq}} \left( \frac{R}{R} \right) \]

-------------------------- (4.12)

\[ \frac{X_{eq}^2}{R} \left( \frac{X_C}{QX R} \right) = \frac{L_{eq}}{QX_C} \]

-------------------------- (4.13)
Where $Q = \frac{X_L}{R} = \frac{X_C}{R}$ and $R << X_{\text{eq}}$. Keep in mind that the reactances in this equation are computed at the resonant frequency. $Q$ often is known as the quality factor of a resonant circuit that determines the sharpness of the frequency response. $Q$ varies considerably by location on the power system. It might be less than 5 on a distribution feeder and more than 30 on the secondary bus of a large step-down transformer. It is clear that during parallel resonance, a small harmonic current can cause a large voltage drop across the apparent impedance, i.e., $V_p = QX_{\text{eq}}I_h$. The voltage near the capacitor bank will be magnified and heavily distorted.
Let us now examine current behavior during the parallel resonance. Let the current flowing in the capacitor bank or into the power system be $I_{\text{resonance}}$; thus,

\[
I_{\text{resonance}} = \frac{V_p}{X_C} + \frac{QX_C I_h}{X_C} + QI_h
\]

or

\[
I_{\text{resonance}} = \frac{V_p}{X_{\text{Leq}}} + \frac{QX_{\text{Leq}} I_h}{X_{\text{Leq}}} + QI_h
\]

\[-\hspace{1cm}-(4.14)\]

\[-\hspace{1cm}-(4.15)\]
From the above equation, it is clear that currents flowing in the capacitor bank and in the power system (i.e., through the transformer) will also be magnified $Q$ times. These phenomenon’s will likely cause capacitor failure, fuse blowing, or transformer overheating.

The extent of voltage and current magnification is determined by the size of the shunt capacitor bank. Figure 4.25 shows the effect of varying capacitor size in relation to the transformer on the impedance seen from the harmonic source and compared with the case in which there is no capacitor.

![Figure 4.25 System frequency response as capacitor size is varied in relation to transformer.](image)

4.7.2 Series Resonance

There are certain instances when a shunt capacitor and the inductance of a transformer or distribution line may appear as a series $LC$ circuit to a source of harmonic currents. If the resonant frequency corresponds to a characteristic harmonic frequency of the nonlinear load, the $LC$ circuit will attract a large portion of the harmonic current that is generated in the distribution system. A customer having no nonlinear load, but utilizing power factor correction capacitors, may in this way experience high harmonic voltage distortion due to neighboring harmonic sources. This situation is depicted in Fig. 4.26
During resonance, the power factor correction capacitor forms a series circuit with the transformer and harmonic sources. The simplified circuit is shown in Fig.4.27. The harmonic source shown in this figure represents the total harmonics produced by other loads. The inductance in series with the capacitor is that of the service entrance transformer. The series combination of the transformer inductance and the capacitor bank is very small (theoretically zero) and only limited by its resistance.

**Figure 4.26** System with potential series resonance problems.

Thus the harmonic current corresponding to the resonant frequency will flow freely in this circuit. The voltage at the power factor correction capacitor is magnified and highly distorted. This is apparent from the following equation:

\[
V_s (\text{at power factor capacitor bank}) = \frac{X_C}{X_T} \frac{V_h}{R} \frac{X_c}{R} V_h
\]

Where \( V_h \) and \( V_s \) are the harmonic voltage corresponding to the harmonic current \( I_h \) and the voltage at the power factor capacitor bank, respectively.

The negligible impedance of the series resonant circuit can be exploited to absorb desired harmonic currents. This is indeed the principle in designing a notch filter. In many systems with potential series resonance problems, parallel resonance also arises due to the circuit topology.
One of these is shown in Fig. 4.27 where the parallel resonance is formed by the parallel combination between $X_{\text{source}}$ and a series between $X_T$ and $X_C$. The resulting parallel resonant frequency is always smaller than its series resonant frequency due to the source inductance contribution. The parallel resonant frequency can be represented by the following equation:

$$h_p = \frac{X_C}{X_T + X_{\text{source}}}$$

\[------------------------ (4.17)\]

4.7.3 Effects of Resistance and Resistive Load

Determining that the resonant harmonic aligns with a common harmonic source is not always cause for alarm. The damping provided by resistance in the system is often sufficient to prevent catastrophic voltages and currents. Figure 4.28 shows the parallel resonant circuit impedance characteristic for various amounts of resistive load in parallel with the capacitance.
As little as 10 percent resistive loading can have a significant beneficial impact on peak impedance. Likewise, if there is a significant length of lines or cables between the capacitor bus and the nearest upline transformer, the resonance will be suppressed. Lines and cables can add a significant amount of the resistance to the equivalent circuit.

![Diagram](image)

**Figure 4.28** Effect of resistive loads on parallel resonance.

Loads and line resistances are the reasons why catastrophic harmonic problems from capacitors on utility distribution feeders are seldom seen. That is not to say that there will not be any harmonic problems due to resonance, but the problems will generally not cause physical damage to the electrical system components.

The most troublesome resonant conditions occur when capacitors are installed on substation buses, either utility substations or in industrial facilities. In these cases, where the transformer dominates the system impedance and has a high $X/R$ ratio, the relative resistance is low and the corresponding parallel resonant impedance peak is very sharp and high. This is a common cause of capacitor, transformer, or load equipment failure.

While utility distribution engineers may be able to place feeder banks with little concern about resonance, studies should always be performed for industrial capacitor applications and for utility substation applications. Utility engineers familiar with the problems indicate that about 20 percent of industrial installations for which no studies are performed have major operating disruptions or equipment failure due to resonance.
In fact, selecting capacitor sizes from manufacturers’ tables to correct the power factor based on average monthly billing data tends to result in a combination that tunes the system near the fifth harmonic. This is one of the worst harmonics to which to be tuned because it is frequently the largest component in the system.

It is a misconception that resistive loads damp harmonics because in the absence of resonance, loads of any kind will have little impact on the harmonic currents and resulting voltage distortion. Most of the current will flow back into the power source. However, it is very appropriate to say that resistive loads will damp resonance, which will lead to a significant reduction in the harmonic distortion.

Motor loads are primarily inductive and provide little damping. In fact, they may increase distortion by shifting the system resonant frequency closer to a significant harmonic. Small, fractional-horsepower motors may contribute significantly to damping because their apparent $X/R$ ratio is lower than that of large three-phase motors.

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