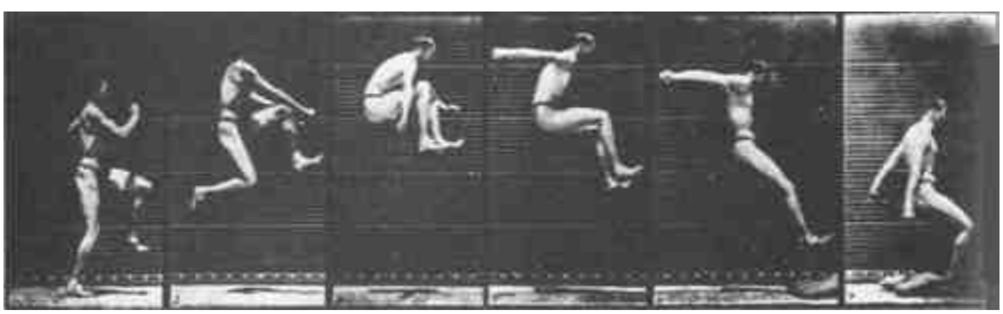
ANGULAR MOMENTUM IN TWO DIMENSIONS

Sure, and maybe the sun won't come up tomorrow.” Of course, the sun only appears to go up and down because the earth spins, so the cliche should really refer to the unlikelihood of the earth's stopping its rotation abruptly during the night. Why can't it stop? It wouldn't violate conservation of momentum, because the earth's rotation doesn't add anything to its momentum. While California spins in one direction, some equally massive part of India goes the opposite way, canceling its momentum. A halt to Earth's rotation would entail a drop in kinetic energy, but that energy could simply be converted into some other form, such as heat.



a / The jumper can't move his legs counterclockwise without moving his arms clockwise. (Thomas Eakins.)

Other examples along these lines are not hard to find. An atom spins at the same rate for billions of years.

A high-diver who is rotating when he comes off the board does not need to make any physical effort to continue rotating, and indeed would be unable to stop rotating before he hit the water.

These observations have the hallmarks of a conservation law:

- *A closed system is involved.* Nothing is making an effort to twist the earth, the hydrogen atom, or the high-diver. They are isolated from rotation-changing influences, i.e., they are closed systems.
- *Something remains unchanged.* There appears to be a numerical quantity for measuring rotational motion such that the total amount of that quantity remains constant in a closed system.
- *Something can be transferred back and forth without changing the total amount.* In the photo of the old-fashioned high jump, a, the jumper wants to get his feet out in front of him so he can keep from doing a “face plant” when he lands. Bringing his feet forward would involve a certain quantity of counterclockwise rotation, but he didn't start out with any rotation when he left the ground. Suppose we consider counterclockwise as positive and clockwise as negative. The only way his legs can acquire some positive rotation is if some other part of his body picks up an equal amount of negative rotation. This is why he swings his arms up behind him, clockwise.

What numerical measure of rotational motion is conserved? Car engines and old-fashioned LP records have speeds of rotation measured in rotations per minute (r.p.m.), but the number of rotations per minute (or per second) is not a conserved quantity. A twirling figure skater, for instance, can pull her arms in to increase her r.p.m.'s. The first section of this chapter deals with the numerical definition of the quantity of rotation that results in a valid conservation law.

When most people think of rotation, they think of a solid object like a wheel rotating in a circle around a fixed point. Examples of this type of rotation, called rigid rotation or rigid-body rotation, include a spinning top, a seated child's swinging leg, and a helicopter's spinning propeller. Rotation, however, is a much more general phenomenon, and includes noncircular examples such as a comet in an elliptical orbit around the sun, or a cyclone, in which the core completes a circle more quickly than the outer parts.

If there is a numerical measure of rotational motion that is a conserved quantity, then it must include nonrigid cases like these, since nonrigid rotation can be traded back and forth with rigid rotation. For instance, there is a trick for finding out if an egg is raw or hardboiled. If you spin a hardboiled egg and then stop it briefly with your finger, it stops dead.

But if you do the same with a raw egg, it springs back into rotation because the soft interior was still swirling around within the momentarily motionless shell. The pattern of flow of the liquid part is presumably very complex and nonuniform due to the asymmetric shape of the egg and the different consistencies of the yolk and the white, but there is apparently some way to describe the liquid's total amount of rotation with a single number, of which some percentage is given back to the shell when you release it.

The best strategy is to devise a way of defining the amount of rotation of a single small part of a system. The amount of rotation of a system such as a cyclone will then be defined as the total of all the contributions from its many small parts.

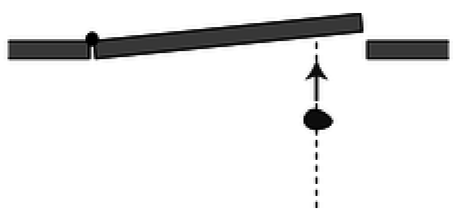


Figure b: An overhead view of a piece of putty being thrown at a door. Even though the putty is neither spinning nor traveling along a curve, we must define it has having some kind of "rotation" because it is able to make the door rotate.

The quest for a conserved quantity of rotation even requires us to broaden the rotation concept to include cases where the motion doesn't repeat or even curve around.

If you throw a piece of putty at a door, (Figure b), the door will recoil and start rotating. The putty was traveling straight, not in a circle, but if there is to be a general conservation law that can cover this situation, it appears that we must describe the putty as having had some "rotation," which it then gave up to the door. The best way of thinking about it is to attribute rotation to any moving object or part of an object that changes its angle in relation to the axis of rotation. In the putty-and-door example, the hinge of the door is the natural point to think of as an axis, and the putty changes its angle as seen by someone standing at the hinge, Figure c. For this reason, the conserved quantity we are investigating is called *angular momentum*. The symbol for angular momentum can't be "a" or "m," since those are used for acceleration and mass, so the letter L is arbitrarily chosen instead.

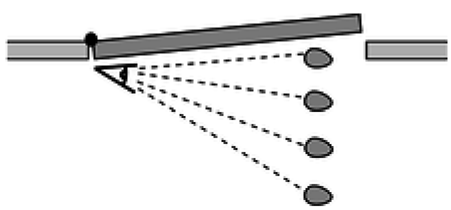


Figure c: As seen by someone standing at the axis, the putty changes its angular position. We therefore define it as having angular momentum.

Imagine a 1 kg blob of putty, thrown at the door at a speed of 1 m/s, which hits the door at a distance of 1 m from the hinge. We define this blob to have 1 unit of angular momentum.

When it hits the door, the door will recoil and start rotating. We can use the speed at which the door recoils as a measure of the angular momentum the blob brought in.[1]

Experiments show, not surprisingly, that a 2 kg blob thrown in the same way makes the door rotate twice as fast, so the angular momentum of the putty blob must be proportional to mass,

$$L \propto m.$$

Similarly, experiments show that doubling the velocity of the blob will have a doubling effect on the result, so its angular momentum must be proportional to its velocity as well,

$$L \propto mv.$$

You have undoubtedly had the experience of approaching a closed door with one of those bar-shaped handles on it and pushing on the wrong side, the side close to the hinges. You feel like an idiot, because you have so little leverage that you can hardly budge the door. The same would be true with the putty blob.

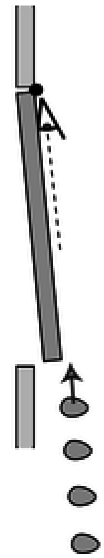


Figure d: A putty blob thrown directly at the axis has no angular motion, and therefore no angular momentum. It will not cause the door to rotate.

Experiments would show that the amount of rotation the blob can give to the door is proportional to the distance, r, from the axis of rotation, so angular momentum must be proportional to r as well,

$$L \propto mvr.$$

We are almost done, but there is one missing ingredient. We know on grounds of symmetry that a putty ball thrown directly inward toward the hinge will have no angular momentum to give to the door. After all, there would not even be any way to decide whether the ball's rotation was clockwise or counterclockwise in this situation.

It is therefore only the component of the blob's velocity vector perpendicular to the door that should be counted in its angular momentum,

$$L=mv_{\perp}r.$$

More generally, $v_{\perp}$ should be thought of as the component of the object's velocity vector that is perpendicular to the line joining the object to the axis of rotation.

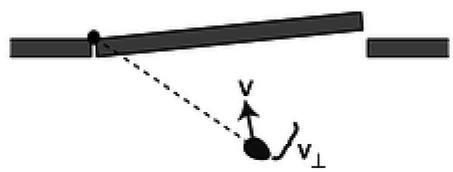


Figure e: Only the component of the velocity vector perpendicular to the line connecting the object to the axis should be counted into the definition of angular momentum.

We find that this equation agrees with the definition of the original putty blob as having one unit of angular momentum, and we can now see that the units of angular momentum are (kg·m/s)·m, i.e., kg·m2/s. Summarizing, we have

$$L=mv_{\perp}r \text{[angular momentum of a particle in two dimensions]},$$

where m is the particle's mass, $v_{\perp}$ is the component of its velocity vector perpendicular to the line joining it to the axis of rotation, and r is its distance from the axis. (Note that r is not necessarily the radius of a circle.) Positive and negative signs of angular momentum are used to describe opposite directions of rotation.

The angular momentum of a finite-sized object or a system of many objects is found by dividing it up into many small parts, applying the equation to each part, and adding to find the total amount of angular momentum. (As implied by the word “particle,” matter isn't the only thing that can have angular momentum. Light can also have angular momentum, and the above equation would not apply to light.)

Conservation of angular momentum has been verified over and over again by experiment, and is now believed to be one of the most fundamental principles of physics, along with conservation of mass, energy, and momentum.

Source:
http://physwiki.ucdavis.edu/Fundamentals/04._Conservation_of_Angular_Momentum/4.1_Angular_Momentum_In_Two_Dimensions