A NOVEL MIXED METHOD FOR THE MODEL ORDER REDUCTION OF LINEAR SYSTEMS USING PARTICLE SWARM OPTIMIZATION AND POLYNOMIAL METHOD

[1]M. SUDHEER KUMAR, [2]N. NAGENDRA & [3]T. MADHUBABU

[1,2,3]Anits engg.college sangivalasa, bheemunipatnam mandal , visakhapatnam,
Email: nagendra.namala@gmail.com, madhubabu202@gmail.com & sudheer269@gmail.com

Abstract - In this paper, a novel mixed method is used for reducing the higher order system to lower order system. The denominator polynomials are obtained by the PSO Algorithm and the numerator coefficients are derived by the polynomial method. This method is simple and computer oriented. If the original system is stable then reduced order system is also stable. The proposed method is illustrated with the help of typical numerical examples considered from the literature.

Keywords *: PSO optimization, polynomial technique, order reduction, transfer function, stability.*

I. INTRODUCTION

The order reduction of a system plays an important role in many engineering applications especially in control systems. The use of reduced order model is to implement analysis, simulation and control system designs. The use of original system is tedious and costly. So to avoid the above problems order reduction implementation is necessary.

Many approaches have been proposed for reducing higher order to lower order system. Shamash [1] proposed a method of reduction using pade approximation. The disadvantage of this method is reduced order model may be unstable even though the original high order system is stable. Hutton and Fried land [3], proposed routh approximation method which is also disadvantage.

One of the most proposing research fields is “Evolutionary techniques” [6]. Among all the Evolutionary techniques particles swarm optimization appears as a promising algorithm. PSO algorithm shares many similarities with the genetic algorithm. one of the most promising advantages of PSO over the GA is its algorithmic simplicity.

In the present paper to overcome above problem a new mixed method is proposed. The denominator polynomials of the reduced order model are obtained by PSO technique and the numerator coefficients are determined by using polynomial technique. The proposed method is compared with the other well known order reduction techniques available in the literature.

PROPOSED METHOD

Let the transfer function of high order original system of the order ‘n’ be

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{a_0+a_1s+a_2s^2+\cdots\ldots\ldots a_{m-1}s^{n-1}}{b_0+b_1s+b_2s^2+\cdots\ldots\ldots b_ns^n} \qquad (1)$$

Let the transfer function of the reduced model of the order ‘k’ be

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{d_0+d_1s+d_2s^2+\cdots\ldots\ldots d_{k-1}s^{k-1}}{e_0+e_1s+e_2s^2+\cdots\ldots\ldots e_ks^k} \qquad (2)$$

A) Determination of denominator by PSO:

The PSO method is a member of wide category of swarm intelligence methods for solving the optimization problems. Particle swarm optimization technique is computationally effective and easier. PSO is started with randomly generated solution as an initial population called particles. Each particle is treated as a point in D dimensional space. Each particle in PSO flies through the search space with an adaptable velocity that is dynamically modified according to its own flying experience and also flying experience of the other particles.

Each particle has a memory and hence it is capable of remembering the best position in the search space ever visited by it. The position corresponding to the best fitness is known as pbest and the overall best of all the particles in the population is called gbest [13].

By using iteration the values of pbest and gbest are calculated. Velocity and particle positions are updated by using below formulae.

$$v_{id}^{k+1} = v_{id}^k + c_1 * r_1 * (p_{id}^k - x_{id}^k) + c_2 * r_2 * (p_{gd}^k - x_{id}^k) \qquad (3)$$

$$x_{id}^{k+1} = x_{id}^k + v_{id}^k \qquad (4)$$

Where ‘v’ is the velocity, ‘x ’is the position, p_{id} and g_{id} are the pbest and gbest, ‘k’ is iteration and c_1,c_2 are the cognition and social parameter. These parameters are variable or constant. Generally these values are ‘2’and r_1, r_2 are the random numbers in the range (0, 1). The parameters c_1 and c_2 determine the relative pull of pbset and gbset and the parameters r_1 and r_2 help in stochastically varying these pulls. To balance the global and local search parameters ‘w’ is introduced, this is inertia weight.

(5)

Where ‘it’ is the number of iteration

(6)

B) Determination of numerator by polynomial method

The numerator polynomial is obtained by equating the original system with reduced order system

=

(7)

Equate the same power’s of‘s’ on both sides, we get

$a_0 e_0 = b_0 d_o$ (8)

$a_0 e_1 + a_1 e_0 = b_0 d_1 + b_1 d_0$ (9)

$a_0 e_2 + a_1 e_1 + a_2 e_0 = b_0 d_2 + b_1 d_1 b_2 d_0$ (10)

$a_{n-1} e_k = b_n d_{k-1}$ (11)

From the above equations we can get the values of d0, d1….d_q.

To match the steady state response multiply the numerator polynomial with ‘K’

NUMERICAL EXAMPLES:

Example1: Consider the order transfer function [6]

G(s)=

(12) For implementing PSO algorithm, to obtained the reduced denominator several parameters are to be considered

The values of c_1 and c_2 are ‘2’

The range of random numbers r_1 and r_2 are (0,1).

Swarm size = 20 (Number of reduced order models)

Unknown coefficients = 2

Number of iterations = 500

The denominator polynomial obtained using PSO algorithm is

$D_2(s)$ = (13)

For finding the numerator values, use the polynomial technique. Equate original transfer function and reduced order transfer function with obtained denominator.

=

(14)

On cross multiplying the above equations and comparing the same power of ‘s’ on the both sides, we get numerator value and multiply the numerator with ‘k.’

Therefore, the numerator by polynomial method is

$N_2(s) = 9.255+5.717s$ (15)

the proposed second order reduced model using mixed method is obtained as follows:

$R_2(s)$ = (16)

the second order reduced model for Mihailov stability criterion and continued fraction technique is [6]

$R_{2a}(s)$ =

(17)

The comparison is made by computing the error index known as integral square error ISE in between the transient parts of the original and reduced order model, is calculated to measure the quality, the smaller the ISE, the closer is R(s) to G(s),which given by;

ISE= dt (18)

Where y(t) and $y_r(t)$ are the unit step responses of original and reduced order systems for a second order reduced respectively. The error is calculated for various reduced order models and proposed method is shown below.

Method	Reduced model	ISE
Proposed mixed method		0.000698
Mihailov stability and continued fraction method		1.0806

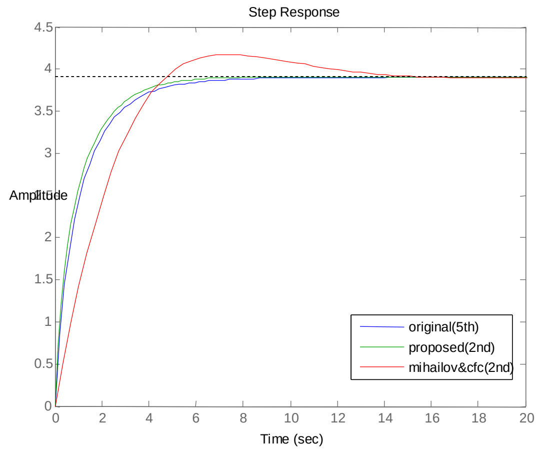

Fig1: Step responses of original and reduced order models

Example2: Let us consider the transfer function [4]

G(s)

(19)

For implementing PSO algorithm, to obtained the reduced denominator several parameters are to be considered

The values of c_1 and c_2 are ‘2’

The range of random numbers r_1 and r_2 are (0,1).

Swarm size = 30 (Number of reduced order models)

Unknown coefficients = 2

Number of iterations = 100

The denominator polynomial obtained using PSO algorithm is

$D_2(s) =$ (20)

For finding the numerator values, use the polynomial technique. Equate original transfer function and reduced order transfer function with obtained denominator.

=

(21)

Cross multiplying above equations and equating with the same powers of 's' we get the numerator values. And multiply the numerator with 'k.' Therefore, the numerator by polynomial method is

$N_2(s) =$ s

The proposed second order reduced model for using mixed method is obtained as follows:

(22)

The second order reduced model for Pade-Routh approximation is

(23)

Method	Reduced model	ISE
Proposed mixed method		0.000236
Pade-Routh method		0.97

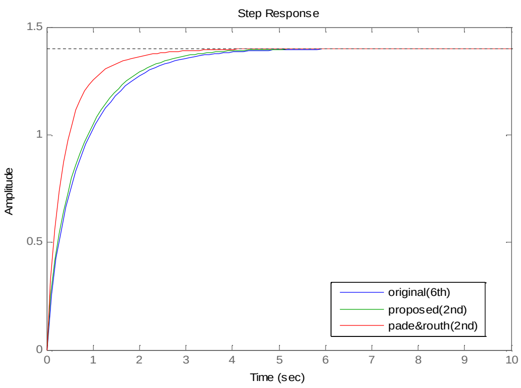

Fig2: Step responses of original and reduced order models

CONCLUSION:

In this paper, a novel mixed method is proposed for reducing a high order system into a lower order system. In this denominator polynomial is obtained by PSO algorithm technique and numerator coefficients are derived by polynomial method. The method has been tested using numerical examples. The results of this method are compared with the other existing methods available in literature shown in fig1 and fig2. From these comparisons, it is concluded that the proposed method is simple; computer oriented and achieves better approximations than the other existing methods. PSO method is based on the minimization of the integral squared error (ISE) between the transient responses of original higher order model and the reduced order model pertaining to a unit step input. This method guarantees stability of the reduced model if the original high order system is stable and which exactly matches the steady state value of the original system.

REFERNCES:

1) Shamash Y "linear system reduction using pade approximation to allow reduction of dominant model int .j .control vol21 no2 pp.257.-272,1975
2) U Singh D Chandra and H .Kar, "improved routh pade approximation a computer aided approach", IEEE trans, autom control 49(2) pp.292-296,2004
3) M.F.Hutton and B.FRIEDLAND, "Routh approximations for reducing order of linear time invariant system" IEEE Trans, auto control vol 20, pp 329-337,1975.
4) Order reduction by error maximization technique by K.RAMESH , A.NIRMAL KUMAR AND G GURUSANELY I EEE 2008 978-1-4244-3595
5) Wan Bai-wu "linear model reduction using mihailov criterion and pade approximation technique" int.J. Control no 33 pp1073-1089, 1981.
6) S.PANDA, S.K.TOMAR, R.PRASAD, C.ARDIL model reduced of linear systems by conventional and evolutionary techniques international journal of computational and mathematical sciences 3:0 2009.
7) Shamash.Y "linear system reduction using pade approximation to allow retention of dominant modes" INT.J. Control vol 21 vol2 pp.257-272, 1975.
8) Shailendra K.MITTAL DINESH CHANDRA and BHARATHI DURIDEVI A computer -aided approach for routh pade approximation of SISO systems based on multi objective optimization IAGSIT int.J. Of engineering and technology vol2, no2, aprl-2010 ISSN: 1993-8236.
9) S.Panda and N.P.PADHY "comparison of particle swarm optimization and genetic algorithm for FACTS- based controller design" applied soft computing. Vol 8, PP 1418-1427,2008
10) Y.Shamesh "model reduction using the routh stability criterion and the pade approximations techniques" int. J. controls Vol21 PP 475-484, 1975.
11) T.C.CHEN, G.Y.CHANG and K.W HAN "model reduction using the stability-equation method and the pade approximation method" journal of franklin institute, vol.304, PP473-490-1980.
12) P.O.GUTMAN, C.F.MANNER FELT and P.MOLANDER "contribution of the model reduction problem IEEE Tran Auto control, vol.27 pp 454-455, 1987.