A COMPARATIVE ANALYSIS OF MEAN SQUARE ERROR ADAPTIVE FILTER ALGORITHMS FOR GENERATION OF MODIFIED SCALING AND WAVELET FUNCTION

RAGHAVENDRA SHARMA*
Department of Electrical Engineering
Dayalbagh Educational Institute
Dayalbagh, Agra, U.P, India-282010
email: raghsharma2000@yahoo.com

V PREM PYARA
Department of Electrical Engineering
Dayalbagh Educational Institute
Dayalbagh, Agra, U.P, India-282010

Abstract:
In this paper, a technique to determine the filter bank coefficients of Wavelets db3 and coif2 using mean square error adaptive filter RLS algorithm has been presented. Filter bank coefficients of the wavelet are treated as weight vector of adaptive filter and are changed iteratively to reach the desired value after few iteration. It is found that of the three adaptive algorithms viz. Least Mean Square (LMS), Normalized Least Mean Square (NLMS) and Recursive Least Square (RLS) algorithm, RLS performs better due to its insensitivity to step size, faster convergence and better accuracy. Modified Scaling and wavelet functions are generated from the filter bank coefficients obtained using RLS algorithm.

Keywords: Filter Bank Coefficients; scaling function; wavelet function; adaptive filter.

1. Introduction
Of late, wavelet analysis is being used in many problems in the area of Digital Signal Processing [1-3]. Prior to the arrival of Wavelet Transform (WT), frequency domain techniques were used for the characterization of the signals [4]. Wavelet is defined as a function of time $\psi(t)$, which is referred to as “mother wavelet”. By stretching and shifting (“dilating and translating”) the “mother wavelet” we obtain “daughter wavelets”. Wavelets are manipulated in two ways. The first one is translation where the position of the wavelet is changed along the time axis. The second one is scaling which implies changing the frequency of the signal. A wavelet $\psi(t)$ can be expressed in terms of $\phi(t)$, the scaling function, as follows;

$$\psi(t) = \sum_{n=-\infty}^{+\infty} g(n) \phi(2t - n)$$  (1)

And the scaling function is defined as,

$$\phi(t) = \sum_{n=-\infty}^{+\infty} h(n) \phi(2t - n)$$  (2)

Where $g(n)$ and $h(n)$ are detail and approximation coefficients respectively, and related as;

$$g(n) = (-1)^{1-n}h(1-n)$$  (3)

A wavelet can be reconstructed from its approximation and detail coefficients [5]. If $h(n)$ is obtained then $g(n)$ is calculated from (3). Most of the wavelet applications are dealt with the coefficients $g(n)$ and $h(n)$ from (1) and (2). These are represented as quadrature mirror filters (QMF) [6][7], having mirror image spectra. The coefficient $h(n)$ has been obtained with the help of various optimization techniques viz. mean square error adaptive filter algorithms LMS and NLMS [8]. An adaptive filter algorithmically alters its parameter iteratively to minimize the mean square error obtained from the difference between the desired value of $h(n)$ and the
actual value. But the values of filter coefficients obtained by these two algorithms are not near to the actual value and also take more time for computation [9]. One of the problems associated with wavelet synthesis is to determine wavelet coefficients accurately, hence in this paper we have used RLS algorithm for determining the filter coefficients \( g(n) \) and \( h(n) \) of the wavelets very precisely along with least time for computation. The modified scaling and wavelet functions are generated with the help of these coefficients obtained with this algorithm. This paper is organized as follows: Theory related to least mean square (LMS), normalized least mean square (NLMS) and RLS algorithm is briefly discussed in section 2. This section compares the differences between the above mentioned algorithms for adaptive filter design. The experimental results using above three algorithms for determining filter bank coefficients of standard wavelets are given in Section 3. Conclusions drawn along with some directions for future research are given in Section 4.

2. Adaptive Filter Algorithms

Adaptive filter algorithms are broadly classified in to the following three categories:

2.1 Least Mean Square (LMS) Algorithm

A uniform linear array with \( N \) isotropic elements, which forms the integral part of the FIR adaptive filter design is shown in [10]. The weights are computed using LMS algorithm based on minimum squared error (MSE), therefore the spatial filtering problem involves estimation of the signal \( s(n) \) from the received signal \( x(n) \) by minimizing the error between the reference signal \( d(n) \), which closely matches or has some extent of correlation with the desired signal estimate and the output \( y(n) \). From the method of steepest decent, the weight vector equation is given by [11];

\[
\mathbf{w}(n + 1) = \mathbf{w}(n) + \frac{1}{2} \mu \nabla \mathbb{E} [e^2(n)]
\]

(4)

Where \( \mu \) is the step size parameter and controls the convergence characteristics of the LMS algorithm; \( e^2(n) \) is the mean square error between the output \( y(n) \) and the reference signal. The LMS algorithm is initiated with an arbitrary value \( w(0) \) for the weight vector eventually leads to the minimum value of the mean squared error. Since \( w(n) \) is a vector of random variables, the convergence of the LMS algorithm [12] should be considered within the statistical framework. For the convergence of the algorithm the step size should satisfy the following condition;

\[
0 < \mu < \frac{2}{\lambda_{\text{max}}}
\]

(5)

Where \( \lambda_{\text{max}} \) is the largest eigenvalue of the correlation matrix. The drawback of the LMS algorithm is its sensitivity to the change in the input signal \( x(n) \), which result in finding difficulty to decide the optimum size of convergence parameter \( \mu \) for convergence of the algorithm along with minimum time of convergence.

2.2 Normalized Least Mean Square (NLMS) Algorithm

Normalized Least Mean Square (NLMS) is actually derived from LMS algorithm. The need to derive NLMS algorithm is that the input signal power changes in time and due to this change, the step size between two adjacent coefficients of the filter will also change, due to which it affects the convergence rate. For weak signals this convergence rate will slow down, and for strong signals convergence rate will be increased, hence producing error. To overcome this problem of convergence rate, we try to adjust the step size parameter with respect to input signal power, therefore the step size parameter is said to be normalized. This algorithm may be a suitable alternative which normalizes the LMS step size with the power of the input signal [13]. When the input signal is too small, the NLMS algorithm can be modified by adding a small positive value \( \delta \) to the power of the input signal. Hence,

\[
w(n + 1) = w(n) + \beta \frac{x'(n)}{\delta + |x(n)|^2} e(n)
\]

(6)

Where \( \beta \) is normalized step size, whose value is \( 0 < \beta < 2 \), and \( \delta \) is safety factor whose value is always lesser than one. So the problem of sensitivity of step size is resolved with NLMS algorithm.

2.3 Recursive Least Square (RLS) Algorithm
The RLS adaptive filter is the time update version of wiener filter [14]. For non stationary signals, this filter tracks the time variations but in case of stationary signals, the convergence behavior of this filter is the same as wiener filter that converges to the same optimal coefficients. This filter has fast convergence rate and it is widely used in various signal processing applications where the signal chances very fast. This adaptive algorithm is computationally complex and has high speed of convergence, minimum error at convergence, numerical stability and robustness. The RLS algorithm is executed in the following way;

1. Choose $\lambda = 0.99$ (always less than 1) and initialize the value of the weight vector as zero, $w(0)=0$; then, the gain vector is given as,

$$K(n) = \frac{\lambda^{-1} P(n-1) u(n)}{1 + \lambda^{-1} u^H(n) P(n-1) u(n)}$$

(7)

2. Compute the error vector given by,

$$e(n) = d(n) - u^H(n) w(n-1)$$

(8)

3. Update the estimate of coefficient value,

$$w(n) = w(n - 1) + K(n) e^*(n)$$

(9)

4. Take the inverse of weighted autocorrelation matrix $P(n)$, given by,

$$P(n) = \lambda^{-1} P(n-1) - \lambda^{-1} K(n) u^H(n) P(n-1)$$

(10)

Each time the value of $n$ is incremented and the steps 1-4 are repeated until the minimum value of the error is achieved.

3. **Experimental Results**

In the algorithm developed, $h(n)$ is tried to be reconstructed using LMS algorithm. Here the desired signal is down sampled and then up sampled before giving to the filter. But it encountered the problem of sensitivity of step size. The obtained $h(n)$ varies with step size as shown in table 1. This problem of sensitivity of step size can be resolved by taking NLMS algorithm in consideration, which is not sensitive to step size as shown in table 2. The number of iterations and the accuracy of the result is improved with RLS algorithm as shown in table 3. The stream of impulses is given as input to the adaptive filter, up-sampling and then passing the estimated output iteratively to the filter helps in calculating the scaling function $h(n)$. The iteration method shown in figure 1 is implemented on coif2 and db3 wavelets.

![Figure 1: Flow chart for scaling function](image)

3.1 **Using LMS Algorithm**

LMS algorithm is implemented for determining the filter bank coefficients of wavelet db3 and coif2. The selection of step size is very crucial in case of LMS algorithm. For wavelet db3, the maximum step size is 0.1662, so if we choose the step size more than this value then the filter coefficients will never converge, and the reconstruction of the wavelet is not possible due to absurd values as shown in the table 1.
3.2 Using NLMS Algorithm

The same experiment is repeated for determination of the filter bank coefficients of db3 with NLMS algorithm which are not sensitive to step size as shown in the table 2.

Table2: NLMS algorithm on wavelet db3

<table>
<thead>
<tr>
<th>Actual h(n)</th>
<th>Step Size</th>
<th>NLMS coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.0498, -0.1208, -0.1909, 0.6504, 1.1411, 0.4705]</td>
<td>0.01</td>
<td>[0.0458, -0.1239, -0.1936, 0.6161, 1.0973, 0.4435]</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>[0.0497, -0.1273, -0.1982, 0.6493, 1.1343, 0.4703]</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>[0.0452, -0.1334, -0.1967, 0.6598, 1.1490, 0.4661]</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>[0.0488, -0.1061, -0.1686, 0.6523, 1.1342, 0.4834]</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>[0.0332, -0.1227, -0.1426, 0.6560, 1.1793, 0.4834]</td>
</tr>
</tbody>
</table>

3.3 Using RLS Algorithm

The same experiment is further carried out for constructing the wavelets db3 and coif2 with RLS algorithm and the reconstruction is successful, with minimum convergence time among the three algorithm and the filter bank coefficients are also much closer to the desired values as shown in the table3.

Table3: RLS algorithm on wavelet db3

<table>
<thead>
<tr>
<th>Actual h(n)</th>
<th>Step Size</th>
<th>RLS coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.0498, -0.1208, -0.1909, 0.6504, 1.1411, 0.4705]</td>
<td>0.01</td>
<td>[0.0497, -0.1246, -0.1876, 0.6486, 1.1416, 0.4704]</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>[0.0463, -0.1249, -0.1958, 0.6451, 1.1401, 0.4689]</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>[0.0470, -0.1244, -0.1917, 0.6478, 1.1363, 0.4690]</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>[0.0497, -0.1230, -0.1910, 0.6488, 1.1442, 0.4709]</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>[0.0461, -0.1226, -0.1988, 0.6445, 1.1374, 0.4682]</td>
</tr>
</tbody>
</table>

The number of iteration used for obtaining the desired values of the filter coefficients of all types of the wavelets with minimum error is shown in the table 4.

Table4: Iterations used by adaptive algorithms

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>No of iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMS</td>
<td>4N</td>
</tr>
<tr>
<td>NLMS</td>
<td>2N</td>
</tr>
<tr>
<td>RLS</td>
<td>N</td>
</tr>
</tbody>
</table>

Where N is the number of filter bank coefficients. This variation in iteration time is very severe in case of filter with larger number of coefficients. The mean square error obtained by RLS algorithm is minimum and settled down very fast as compared to other two algorithms; hence least time for iteration as shown in figure 3. The modified scaling and wavelet functions of the wavelet db3 and coif2 using filter bank coefficients obtained by RLS algorithm are shown in the figures 4, 5, 6 and 7.
Figure 2: Number of iterations for adaptive algorithms

Figure 3: Learning curves for three adaptive filter algorithms on wavelets

Figure 4: New scaling function of db3

Figure 5: New wavelet function of db3

Figure 6: New scaling function of coif2

Figure 7: New wavelet function of coif2
4. Conclusions

LMS, NLMS and RLS are the three popular algorithms for adaptive filter design. Among the three algorithms, NLMS and RLS are very useful due to its quality of insensitivity to step size. These two algorithms can be used for determining the filter bank coefficients of the wavelets. In this paper, NLMS and RLS algorithms are compared to determine the filter bank coefficients of the wavelet, then, RLS algorithm supersedes NLMS in context to mean square error and lesser number of iteration for convergence. So this algorithm has been used to determine the filter bank coefficients of wavelet db3 and coif2 and reconstructed the scaling and wavelet function. The number of iterations used for the convergence of the algorithm for db3 is six only, hence least time for convergence. Wavelets are very useful for the analysis of musical signals; hence, this work can be extended for identification of the wavelet present in the sounds produced by musical instruments.

References