System Identification with Noisy Data

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***Abstract* -** System identification from the field obtained data is not successful till date because of the noise in the sensor or measured. The unknown system parameter for any unknown damage is still a long way to move. In this paper finite element technique is applied on the simulated damaged structures. The damage detection for the noise free data is extended and tried for the random noisy sensor data. The proposed algorithm is used to predict the system parameter on static deflection data with introduction of some random noisy data using simulated structure. The developed algorithm is applied on bridge truss structure for the identification of the damaged system parameter prediction.

Key words - *Damage, System identification; Noisy data; Finite element.*

I. INTRODUCTION

The prediction of the damaged parameters from the field data having noise is still an open challenge in the field of structural identification. The Structural Health Monitoring (SHM) of large space structures like aerospace structure; satellite vehicles, where once the damage had occurred cause the uncertainty due to unknown damaged parameter. Finite element methods are universally accepted as fast computation tools for structural behavior prediction. Development in the field of computational technology and sensing instruments has also progressed a lot. Field data are usually noisy. The damage detection, location and damage extent prediction are among the important aspect of structural behavior prediction. The main aim of researchers is concentrated on the suitable identification algorithms based on uncertain available sensor data which may be linear or nonlinear, noisy or noise free sensor data.

There are two kinds of parameter identification namely static and dynamic parameter identification. In the dynamic parameter identification there are three unknowns. They are mass, stiffness and damping. The relationship between the dynamic coefficients (mass, stiffness and damping) and also its sensitivity effects of one property on another are still unknowns. Such a system makes the analysis very complicated. For uncertain damage with uncertain noisy data, no certain techniques are available. The static identification method seems to be better than dynamic identification as it is having only one unknown (stiffness).

A brief review on damage detection mainly using the static method or static combined with dynamic method is presented here. The static damage parameter identification approaches by the error term reduction includes minimum deviation, sensitivity analysis, output error optimization etc. were approached by [2] , [4], [7], [12], [14] and [15]. Damage detection in composite materials using system identification technique proposed by [14]. The output error approach of system identification was employed to determine the changes in the analytical model in order to minimize the distance between measured and predicted response. [2] Used force error estimator and displacement error estimator for static parameter grouping scheme to identify the error by least squares minimization. Static strain measurement from multiple loading models for identification of the hole and cracks in linear anisotropy elastic materials with nonlinear optimization was presented by [4]. [15] applied a linear constrained nonlinear optimization problem using the minimization of error between the measured and computed displacement to find damage.

Error sensitivity analysis is found to be a popular method for finding the damage existence [9], [11], [13], [1. On analyzing the sensitivity coefficients for natural frequency, mode shapes and modal flexibility, [9] found that modal flexibility is more sensitive as damage indicator. Sensitivity with some other term approach like orthogonal sensitivity, non-linearity, modal updating were approached by[3], [16], [20] etc. The orthogonal condition sensitivities was developed by [16] for the damaged and undamaged structure mode shapes using FEM in laminated rectangular plate of composite structures. [20] presented a sensitivity-based finite element (FE) model updating for damage detection. The modal flexibility residual formulation and its gradient were used for formulation. The damage detection procedure was illustrated on a simulated example with noisy data and on a reinforced concrete beam model.

Dmage detection of cable-stayed bridges by changes in cable forces, was optimized on cable force error between measurement results and analytical model by [17]. [10] developed a method used continuous strain data from fiber optic sensor and neural network model. [18] used sensitive characteristic of strain to identify damage in structures for strain-based damage identification. [8] Used conventional single-objective optimization approach defines the objective function by combining multiple error terms into a single one, for weaker constraint in solving the identification problem.

The above literature review indicates that both static as well dynamic methods were used in the system parameter prediction of structure using simulated, experimental and from the field data. Static methods used either strain or displacement as measured data, while in case of dynamic methods frequency and mode shapes data were used popularly. Sensitivity method, analyses the effect of parameter changes on the other parameters. When the structural parameters are unknown, the sensitivity analysis has no meaning. In addition, noise in the measured data may completely change the matrix property using matrix inversion. The structural parameter identification from the field data has no well-established solution, until now.

The paper is attributed to the static parameter identification process with the noisy sensor data. The objective of this paper is to develop a new modified approach for the noisy sensor data with few measurement. The initial parameter identification algorithm has been taken from [13] but some noise were introduced in the sensor data. This will lead to near realistic field situation. Finite element method for damage detection using static test data for smaller subgroups of matrix was applied [3] for damage existence prediction with only few measurements. The noise values were varied between ± 4 % errors in the sensor data. A finite element model of bridge truss structure presented for the demonstration.

II. INITIAL AND DEDELOPED APPROACH

An algorithm to find the parameter extent for noise free data was developed by [13]. Assuming the structure behavior of structure is linear throughout the test, the force displacement relationship in the static case for undamaged structure is given by

$$[F] = [K][U] \tag{1}$$

and for damaged structure by

$$[F] = [K_d][U] \tag{2}$$

Partitioning into measured and unmeasured displacements

$$\begin{bmatrix} \boldsymbol{f_a} \\ \hline \boldsymbol{f_b} \end{bmatrix} = \left[\begin{array}{c|c} \boldsymbol{K_{daa}} & \boldsymbol{K_{dab}} \\ \hline \boldsymbol{K_{dab}} & \boldsymbol{K_{dbb}} \end{array}\right] \begin{bmatrix} \boldsymbol{U_a} \\ \hline \boldsymbol{U_b} \end{bmatrix} \tag{3}$$

$$[f_a] = [[K_{aa}] - [K_{ab}]^{-1}[K_{bb}]^{-1}[K_{ab}]]\,[U_a] + [K_{ab}][K_{bb}]^{-1}[f_b] \tag{4}$$

$[f_a]$, $[f_b]$ and $[U_a]$ are measured from test. The difference between the measured and theoretically calculated value will be the error term. If the stiffness parameters are correct, then error matrix $[E(p)]$ will be zero, otherwise non-zero.

$$[E(p)] = [[K_{aa}] - [K_{ab}]^{-1}[K_{bb}]^{-1}[K_{ab}]]\,[U_a] + [K_{ab}][K_{bb}]^{-1}[f_b] - [f_a] \tag{5}$$

$$[E(p)] \approx \{E(p)\} + \{S(\delta p)\}[\Delta p] \tag{6}$$

The error sensitivity expression was calculated using first order Taylor series expansion of error matrix $[E(p)]$. The stiffness parameters were obtained by minimization of error function with respect to unknown parameter ***(p)*** using the least square optimization.

The error sensitivity expression has been modified for noisy sensor data as the displacement gets modified due to sensor noise and unknown damaged. The unknown parameters become the function of both displacements. The sensitivity matrix was recalculated with this new modified expression,

$$[S(p_i)] = [\frac{\partial[k_{aa}]}{\partial p_i} - \frac{\partial[k_{ab}]}{\partial p_i}[k_{bb}]^{-1}[k_{ba}] - [k_{ab}][k_{bb}]^{-1}\frac{\partial[k_{ab}]}{\partial p_i}$$

$$+[k_{ab}][k_{bb}]^{-1}\frac{\partial[k_{ab}]}{\partial p_i}[k_{bb}]^{-1}[k_{ba}]\ \ ][U_a] + \left[[k_{aa}] - [k_{ab}][k_{bb}]^{-1}[k_{ab}]\right]\frac{\partial[U_a]}{\partial p_i}$$

$$+\left[\frac{\partial[k_{ab}]}{\partial p_i}[k_{bb}]^{-1} - [k_{ab}][k_{bb}]^{-1}\frac{\partial[k_{bb}]}{\partial p_i}[k_{bb}]^{-1}\right]*$$

$$\left[[f_a] - \left[[K_{aa}] - [K_{ab}]^{-1}[K_{bb}]^{-1}[K_{ab}]\right][U_a]\right]*[K_{ab}]^{-1}[K_{bb}]$$

$$+\left[[K_{ab}][k_{bb}]^{-1}[f_b] - [f_b]\right]\left[[f_a] - \left[\frac{\partial[k_{aa}]}{\partial p_i}[k_{bb}]^{-1} + [K_{aa}]\frac{\partial[k_{aa}]}{\partial p_i} - \right][U_a]\right]*[K_{ab}]^{-1}[K_{bb}]$$

$$+\left[[f_a] - \left[\frac{\partial[k_{aa}]}{\partial p_i}[k_{bb}]^{-1} + [K_{aa}]\frac{\partial[k_{aa}]}{\partial p_i} - \right]\left[\frac{\partial[U_a]}{\partial p_i}\right]\right]*[K_{ab}]^{-1}[K_{bb}] \quad (7)$$

The present method is able to predict the damage existence in structure with only few measurements. Using least square optimization technique, all the unknown damaged parameters can be identified after some iteration successfully with the noisy sensor data. The finite element method revisited and a row-echelon form of matrix developed for damage detection using static displacement and force data, Dewangan U. K. (2010) [4]. The row-echelon form has an advantage of partitioning matrix into smaller subgroups.The noise values were varied between ± 4 % errors in the sensor data. A finite element model of bridge truss structure presented for the demonstration. The modified algorithm is implemented using MATLAB [21].

III. EXAMPLES

Based on the algorithm discussed in the above section, the computer program was written in MATLAB and tested Bridge Truss Structure Bridge Truss Structure

Structural Details: All elements are having the modulus of elasticity $E = 210\,GPa$ and initial undamaged cross sectional area $A = 1.61 X\, 10^5\ mm^2$. The structural configuration is shown in Fig. 3.8. A single concentrated load is applied at each joint. The deflections at each joint are measured. The force matrix $[F]$ is found out and corresponding displacement matrix $[U]$ is measured.

For bridge truss structure, [1] as shown in Figure 1, the modulus of elasticity of all elements was 206.8 GPa and initial undamaged cross sectional area of all members was 500 mm^2. The noise was introduced up to ± 5 % error in the sensor data for sensor numbers 5, 6, 7 and 8 displacement d.o.f. Different load combinations are considered and they are tabulated in the Table 2 with sensor noise value. Previously discussed algorithms were applied to this problem and results are given in Table 2 and Figure 4 for typical cases. For the noisy sensor data set combination on tower truss the computed parameter values were compared with the actual parameter value. From the plotted graph as shown in Figure 2, it is clear that with the noisy sensor data, [13] algorithm values are far away from the actual value of the parameter. The modified proposed algorithm values are nearly close to the actual value of the parameters. The algorithm could identify the damage extent for members away from the supports.

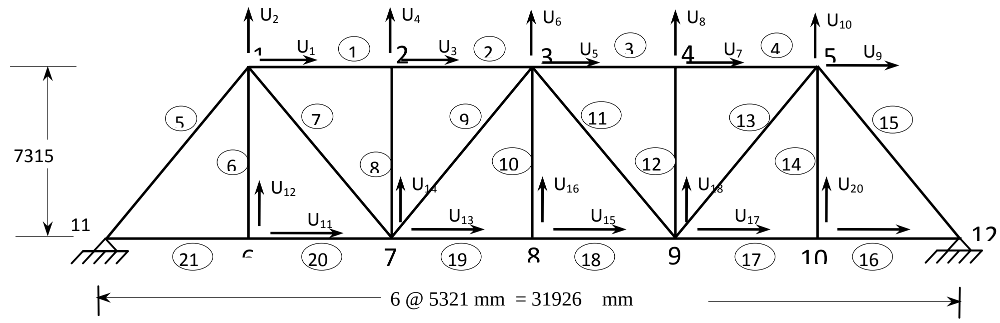

Fig. 1 : Railway bridge truss

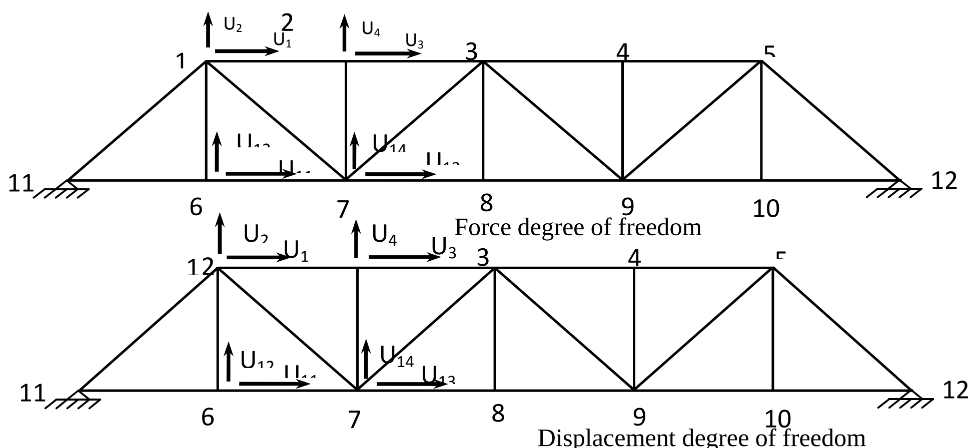

Figure 2 Railway bridge truss (Case 2)

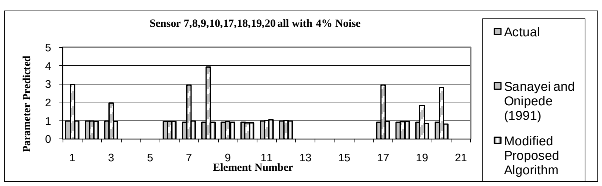

Fig. 3 : Comparative study of damage extent prediction (Bridge truss Structure)

TABLE I: RESULTS OF NOISY SENSOR DATA WITH MODIFIED ALGORITHM ON BRIDGE TRUSS

Case	Member damaged	Applied force d.o.f.	Measured displacement d.o.f.	Noise in Sensor	Members Converged	Members Diverged
1	All	1-20	1-20	8 up to 5%	1-4, 6-14, 17-20	5,16,15 and 21
2	All	7-10, 17-20	7-10, 17-20	1- 8 up to 5%	1-4, 6-14, 17-20	5,16,15 and 21
3	All	7-10, 17-20	1-4, 11-14	4 up to 5%	1-4, 6-14, 17-20	5,16,15 and 21
4	All	1-20	1-20	5,6,7 and, 8 up to 4%	1-4, 6-14, 17-20	5,16,15 and 21

IV. RESULTS

Considering the various multiple damaged member combinations, the existence of damage is detected.

Different combinations of force DOF and displacement DOF used are given in Table 1for various cases.

From the plotted graph as shown in Figure 4, it is clear that with the noisy sensor data, the modified proposed algorithm values are nearly close to the actual value of the parameters. The algorithm could identify the damage extent for members away from the supports.

V. DISCUSSION

In this study, it was clearly observed from the demonstrated examples that the modified algorithm could predict damage extent in the presence of noise in structural response. Further it was found that the algorithms failed to predict damage extent in the members, which were near to the support. Hence, the present work is very well justified for the large space structures where except the support connected members, all other members, damage extent could be predicted. The failure of the algorithm in identifying the damage extent in members near support is a matter of further investigation.

VI. CONCLUSION

The damage parameter identification with the noisy sensor data was carried out for different random noise value. Algorithm developed by [13] has been modified for the noise in sensor value. Finite element method for damage detection using static test data for smaller subgroups of matrix was applied [3] for damage existence prediction with only few measurements. The noise values were varied between ± 4 % errors in the sensor data. The algorithm works well for lower noise level up to a value ± 4 % errors in the sensor data and the unknown damaged parameter could be extracted even from the noisy data set of structural response using above technique for the members with only few measurements. Hence the algorithm is useful for damage prediction with noisy sensor data on large structures for the members, which are away from the supports.

NOTATION

d.o.f. = Degree of freedom

d.d.o.f. = Displacement degree of freedom

f.d.o.f. = Force degree of freedom

{E(p)} = Error function vector

[Fa] = Applied force matrix at force d.o.f. at measured d.o.f.

[Fb] = Unapplied force matrix at force d.o.f. for unmeasured displacement d.o.f.

F.E.M. = Finite element method

[K] = Undamaged global stiffness matrix

[Kdaa] = Sub matrix of [K] corresponding to measured d.o.f. and applied force d.o.f.

[Kdab] = Sub matrix of [K] corresponding to measured d.o.f. and unapplied force d.o.f.

[Kdbb] = Sub matrix of [K] corresponding to unmeasured d.o.f. and unapplied force d.o.f.

[Kdba] = Sub matrix of [K] corresponding to unmeasured d.o.f. and applied force d.o.f.

[Kd] = Damaged global stiffness matrix

n = Number of elements

p = Unknown parameter’s values

r = Row number

[Sd] = Damaged global stiffness matrix

[S] = Undamaged global stiffness matrix

[Saa] = Sub matrix of [S] corresponding to measured d.o.f. and applied force d.o.f.

[Sab] = Sub matrix of [S] corresponding to measured d.o.f. and unapplied force d.o.f.

[Sbb] = Sub matrix of [S] corresponding to unmeasured d.o.f. and unapplied force d.o.f.

[Sba] = Sub matrix of [S] corresponding to unmeasured d.o.f. and applied force d.o.f.

{S(p)} = Sensitivity matrix

[U] = Transformation matrix

[Ua] = Measured displacements matrix

[Ub] = Unmeasured displacements matrix

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