STRUCTURAL DAMAGE EXISTENCE PREDICTION WITH FEW MEASUREMENTS

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Abstract:
Inverse problem of the structural system identification from the field data is still an open challenge. When the structural parameters are unknown, the sensitivity analysis has no meaning. In addition, noise in the measured data may completely change the matrix property using matrix inversion. In this paper, a novel approach to identify the structural damage existence is present for the damage existence identification. The distributed stiffness parameter finite element model is used instead of the lumped parameter model. The present method is able to predict the damage existence in structure with only few measurements even noise at other locations does not affect the parameter identifications. A finite element model of a tower truss structure presented for the demonstration.

Key Words:  System identification; inverse problem; damage detection.

1. Introduction

Structural Health Monitoring (SHM) of Civil engineering structures like building, bridges, offshore oil platform, large space structures and satellite vehicles are not only important from the high capital cost point of view but also important from the safety implementation point of view for a developed society. The non-destructive techniques are common for damage detection. The system parameter identification uses the global responses of field data. The finite elements technique universally accepted as a fast computation tool for the development of damage detection, structural control and system identification. The development of a robust universal computational technique is the need of the field response data.

The damage existence detection, location and extent predictions are among the important aspect of structural behavior prediction from the field data. Existence of damage is usually determined by comparing the changes in structural parameters with respect to its original state. It can be a simulated computer model, experimental model or a real field data. If there is any adverse change in the structural response, the structure said to be damage. A brief review of damage detection using inverse identification problem based on the finite element technique containing theoretical and experimental part on damage detection mainly using the static method or static combined with dynamic methods are presented here.

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The static damage parameter identification approaches by the error term reduction includes minimum deviation, sensitivity analysis, output error optimization etc. The approached are reported elsewhere [Banan et al. (1994)], [Hwu and Liang (2001)], [Wang et al. (2001)], [Paola and Bilello (2004)]. force error estimator and displacement error estimator for static parameter grouping scheme to identify the damage error by least squares minimization was presented by [Banan et al. (1994)]. [Hwu and Liang (2001)] used static strain measurement from multiple loading models for identification of the hole and cracks in linear anisotropy elastic materials with nonlinear optimization. [Paola and Bilello (2004)] proposed a damage identification procedure for Euler-
Bernoulli beams under static loads. An identification procedure presented, based on a least-square constrained nonlinear minimization problem.

Error sensitivity analysis is one of the popular method for finding the damage existence [Zhao and Dewolf (1999)], [Yam et al. 2002], [Hual et al. (2009)]. [Zhao and Dewolf (1999)] found modal flexibility based on the lowest three modes indicate changes in the structural performance when changes occurred in the end bearings of a two span continuous bridge. While analyzing the sensitivity coefficients for natural frequency, mode shapes and modal flexibility, they found that modal flexibility is more sensitive as damage indicator. [Sanayei and Onipede(1991)] measured static displacement data with different sets of load and displacements combinations. Damage modeled as stiffness reduction in the cross-sectional area. Authors were able to find the damage parameters in the members of simulated structure with noise free data. [Yam et al. (2002)] proposed sensitivities analysis in static and dynamic parameters damage indices quantification for their identification capabilities over plate-like structures. [Hual et al. (2009)] found damage detection of cable-stayed bridges by changes in cable forces, they optimization cable force error between measurement results and analytical model.

Sensitivity with some other term approach like orthogonal sensitivity, non-linearity, modal updating were approached by [Vanlanduit et al. (2003)]; [Jaishi and Ren (2006)] etc. [Vanlanduit et al. (2003)] combined the linear and non-linear method of damage detection in static and dynamic method on damage detection of beam. [Jaishi and Ren (2006)] presented a sensitivity-based finite element (FE) model updating for damage detection. The modal flexibility residual formulation and its gradient used for formulation. Lee et al., developed a method used continuous strain data from fiber optic sensor and neural network model.

The probabilistic and statistical approaches were presented by [Hjelmstad and Shin, (1997)], [Sohn and Law (2001)] and many others. [Hjelmstad and Shin, (1997)] proposed a data perturbation scheme for the baseline structure, to establish the damage threshold between noise and the damaged structure to compare the damage indices. Damage characterized as reduction in constitutive property of parameter in FEM model between two time-separated interferences. [Sohn and Law (2001)] used extracted Ritz vectors for damage detection of a grid-type bridge structure using a Bayesian probabilistic approach.

The above literature review indicates that both static as well dynamic methods used in the damage existence prediction of structure using simulated, experimental and from the field data. Static methods used either strain or displacement as measured data, while in case of dynamic methods frequency and mode shapes data were used popularly. Sensitivity method, analyses the effect of parameter changes on the other parameters. When the structural parameters are unknown, the sensitivity analysis has no meaning. In addition, noise in the measured data may completely change the matrix property using matrix inversion. The structural parameter identification from the field data has no well-established solution, until now.

In this paper, a novel approach to identify the structural damage existence is present for the inverse problem. The distributed stiffness parameter is use instead of the lumped parameter. Inverse system identifications using matrix inversion as a whole and sensitivity analysis discussed. In this paper, the finite element method revisited and a row-echelon form of matrix developed for damage detection using static displacement and force data. The row-echelon form has an advantage of partitioning matrix into smaller subgroups. The present method is able to predict the damage existence in structure with only few measurements. The noise response at other locations does not affect the parameter identifications. The developed approach is tests on tower truss structures model.

2. Theoretical Background

In order to detect small flaw or small damage, the data should, never be averaged. Damage detection algorithm needs to develop accurate prediction with perfection of small damages at early age. The damage may be a small change due to small flaws. This change may be very small and may be in one or more locations.

Two matrices will be the equal if \( K_{ij} = K_{dij} \). It is possible when each element of matrix/structure is undamaged. If the matrix changes, (i.e. \( K_{ij} \neq K_{dij} \)), then structure is said to be damaged. Here \( K_{ij} \) is the global stiffness metric, which reduces to \( K_{dij} \) when damage existence occurs at any location.
Similarly, damage due to other changes like changes in the material and geometric properties can also be expressed. In the matrix inversion operation $A^{-1}$, $a_{ij}$ is replaced by all the other elements except $a_{ij}$. In matrix, inverse element $a_{ij}$ is replaced by its cofactor called as $ijA$. The interesting point is that the particular element $a_{ij}$ always left out in the matrix inversion. In other words, in the inversion the damaged matrix, the element $a_{ij}$ always replaced by other parameter, that may be undamaged but its property changes completely.

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}$$

Fig. 2. Matrix inversion

For example, $A_{11}$ is cofactor of $a_{11}$ while finding inverse of $a_{11}$, it is replaced by other elements those are having no contribution in the property of the element $a_{11}$. Although indirectly, they have the contribution in the determinate of $A$. Therefore, the matrix inversion must be avoided in small damage existence prediction. In the process of parameter identification, using inverse technique the matrix inversion changes the property of damaged element.

Static parameter identification was carried out, [Sanayei and Onipede (1991)], on structure by applying a set of force $F$ and the corresponding measured displacements set $U$. Stiffness matrix of undamaged structure is denoted by $[K]$. Let the damaged stiffness of the structure be $[K_d]$. If there is any change in the structural response (static displacement), the structure is said to be damaged. It is assumed that the undamaged structural stiffness parameters are known and the FEM is well developed to represent the structural static response. It may have single or even multiple damaged members. Assuming that the behavior of structure is linear throughout the test, the force displacement relationship in the static case for undamaged structure is given by

$$[K][U] = [F] \quad (1)$$

and for damaged structure by

$$[K_d][U_d] = [F] \quad (2)$$

Partitioning into measured and unmeasured displacements

$$\begin{bmatrix} K_{da} & K_{db} \\ K_{da} & K_{db} \end{bmatrix} \begin{bmatrix} U_{da} \\ U_{db} \end{bmatrix} = \begin{bmatrix} F_{da} \\ F_{db} \end{bmatrix} \quad (3)$$

$[F_{da}], [F_{db}], [U_{da}], [U_{db}]$ are known, while the stiffness parameters are unknown. The damaged stiffness matrix $[K_d]$ is calculated from using the Eq. 2. The matrix partition Eq. 3 can be applied in order to find damage in a sub group with smaller sets of forces.

3. Development of a New Approach

The problem in the above matrix inversion is that the complete inversion of displacement matrix is required. Practically it is very difficult to measure the displacement at all degree of freedom point. The complete matrix
inversion is not possible for a field structure. Also in the matrix inversion process, the noise at other measured locations will change the property of the inverse matrix completely as stated above. Hence the total inversion of matrix method is facing the problem on the field structural parameter identification from the measured field and it is unsuccessful till date.

A new method is proposed for damage detection using row-echelon form of matrix. A Row-echelon form of matrix is used for finding matrix inverse. In the following paragraph, row-echelon form of matrix is described.

The generalized inverse of displacement matrix is obtained here with the help of row-echelon Form. A matrix is said to be in row-echelon form, which has the following properties:

- If a row does not consist entirely of zeros then the first non-zero entry in the row is a 1 (this is called a leading 1).
- If neither row \( i \) or row \( (i + 1) \) consist entirely of zeros the leading 1 in row \( (i +1) \) occurs further to the right than the leading 1 in row \( i \).
- If there are any rows consisting entirely of zeros then they are grouped together at the bottom of the matrix. All the elements in the same column lying under a leading 1 must be 0.

The matrix is said to be in reduced row-echelon form if all the entries lying below leading 1’s are also equal to 0.

In MATLAB code, [Pratap(2002)] row-echelon form to calculate generalized inverse of a matrix \([U]\) is given by command \( \text{rref}([U \ F]) \). This function is modified for two matrices namely displacement and force. The generalized matrix inversion for stiffness matrix is computed using a new modified form of row-echelon function

\[
K = \text{rref}([U \ F])
\]  

(4)

Where, matrix \([U]\) is the field-measured data. This new function also works well with the few numbers of forces when applied to a small portion of structure. The advantage of row-echelon form is that it can be used with lesser number of applied forces in order to find inverse of the small sub-group. Advantage of this inversion is that a part of the structure can be used in order to check the existence of damage. The formula is very simple and it is in a generalized form. This row-echelon function is tested on different structure on simulated noise free data in the following paragraphs.

3.1. Demonstrative examples

The numerical example of the tower truss structure [Sanayei and Onipede(1991)] , is used for demonstration purpose. The tower truss structure is as shown in Fig. 3. The modulus of elasticity of all elements is 206.8 GPa and initial undamaged cross sectional area of all members is 322.6 mm².
Assuming that the load and deflections are small and the structural behavior is linear throughout the test. The force matrix $[F]$ is a unit matrix with its magnitude equal to applied force. For this study a single concentrated load is applied, one at a time, at the force degree of freedom (DOF) and all corresponding deflections are measured at displacement DOF. The value of load is kept constant for a set of the test. The load position is changed to another force DOF and corresponding displacements are measured again for new load position at all displacement DOF. Hence for the one set of load the force matrix generated which is a unit matrix whose magnitude is kept outside.

The stiffness matrix is calculated as per “Eq. (4)” using the row-echelon code, using force and displacement values. Stiffness matrix has been also calculated from linear finite element model. The total stiffness matrix $[K]$ is matrix of 12 X 12 size. But for simple understanding, only 8 X 8 partitioned portion of the matrix is shown here. The matrix is calculated using the matrix partitioning into $[U_m]$ of size [(1:8),(1:8)] and $[F_m]$ of size [(1:8),(1:8)]. The stiffness matrix for 1 to 8 DOF along row and column $[K_m]$ is calculated as shown below.

$$F_{eq} = \begin{bmatrix}
100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 100 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 100 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 100 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 100 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 100 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 100 \\
\end{bmatrix}$$

All dimensions are in mm.

Fig. 3. Tower truss structure
In case if it is required, to find stiffness matrix for a subgroup of says 5 X 5 size only. This can be found out using the row-echelon code “Eq. (4)”. The corresponding force, displacement and the stiffness matrix is calculated and shown below.

\[
U_{aa} = \begin{bmatrix}
0.0011 & -0.0001 & 0.0007 & 0.0001 & 0.0008 & 0 & 0.0008 & 0.0002 \\
-0.0001 & 0.0009 & -0.0001 & 0 & 0.0004 & 0.0007 & 0.0004 & 0.0002 \\
0.0007 & -0.0001 & 0.0011 & 0.0001 & 0.0008 & -0.0002 & 0.0008 & 0 \\
0.0001 & 0 & 0.0001 & 0.0009 & -0.0004 & 0.0002 & -0.0004 & 0.0007 \\
0.0008 & 0.0004 & 0.0008 & -0.0004 & 0.0023 & 0.0002 & 0.02 & -0.0003 \\
0 & 0.0007 & -0.0002 & 0.0002 & 0.0014 & 0.0003 & 0.0003 & 0.0003 \\
0.0008 & 0.0004 & 0.0008 & -0.0004 & 0.002 & 0.0003 & 0.0023 & -0.0002 \\
0.0002 & 0.0002 & 0 & 0.0007 & -0.0003 & 0.0003 & -0.0002 & 0.0014 \\
\end{bmatrix}
\]

\[
K_{aa} = \begin{bmatrix}
19.1438 & 8.4791 & -7.5000 & 0.0000 & -1.1859 & -3.5576 & -4.8000 & -3.6000 \\
8.4791 & 25.1634 & 0.0000 & 0.0000 & -3.5576 & -10.6727 & -3.6000 & -2.7000 \\
-7.5000 & 0.0000 & 19.1438 & -8.4791 & -4.8000 & 3.6000 & -1.1859 & 3.5576 \\
0.0000 & 0.0000 & -8.4791 & 25.1634 & 3.6000 & -2.7000 & 3.5576 & -10.6727 \\
-1.1859 & -3.5576 & -4.8000 & 3.6000 & 18.4859 & -0.0424 & -12.5000 & 0.0000 \\
-3.5576 & -10.6727 & 3.6000 & -2.7000 & -0.0424 & 13.3727 & 0.0000 & 0.0000 \\
-4.8000 & -3.6000 & -1.1859 & 3.5576 & -12.5000 & 0.0000 & 18.4859 & 0.0424 \\
-3.6000 & -2.7000 & 3.5576 & -10.6727 & 0.0000 & 0.0000 & 0.0424 & 13.3727 \\
\end{bmatrix}
\]
Even the stiffness matrix of 2 X 2 sizes can be obtained using the row-echelon form of reduced order force matrix and displacement matrix as discussed above steps. The above matrix parameters are compared to \[ K \] of size 8 X 8; the stiffness parameters are exactly the same in both cases. It is concluded that even with small size of observations of displacements, the stiffness parameters can be computed accurately.

The force matrix \[ F_{aa} \] is a unit matrix with its magnitude equal to applied force. For this study, a single concentrated load is applied, one at a time, at the force DOF and all corresponding deflections are measured at displacement DOF. The value of load is kept constant for a set of the test. The load position is changed to another force DOF and corresponding displacements are measured again for new load position at all displacement DOF. The simulated damaged stiffness matrix is calculated from “Eq. (4)” using row-echelon Form:

\[
K_{aa} = \begin{bmatrix}
19.1438 & 8.4791 & -7.5000 & 0.0000 & -1.1859 \\
8.4791 & 25.1634 & 0.0000 & 0.0000 & -3.5576 \\
-7.5000 & 0.0000 & 19.1438 & -8.4791 & -4.8000 \\
0.0000 & 0.0000 & -8.4791 & 25.1634 & 3.6000 \\
-1.1859 & -3.5576 & -4.8000 & 3.6000 & 18.4859 
\end{bmatrix}
\]

\[
U_{aa} = 1 \times 10^{-3} X
\begin{bmatrix}
0.7891 & -0.2549 & 0.3805 & 0.1171 & 0.0775 \\
-0.2549 & 0.4918 & -0.1043 & -0.0437 & 0.0597 \\
0.3805 & -0.1043 & 0.8265 & 0.2543 & 0.1694 \\
0.1171 & -0.0437 & 0.2543 & 0.4873 & -0.0298 \\
0.0775 & 0.0597 & 0.1694 & -0.0298 & 0.6072
\end{bmatrix}
\]

Even the stiffness matrix of 2 X 2 sizes can be obtained using the row-echelon form of reduced order force matrix and displacement matrix as discussed above steps. The above matrix parameters are compared to \[ K_{aa} \] of size 8 X 8; the stiffness parameters are exactly the same in both cases. It is concluded that even with small size of observations of displacements, the stiffness parameters can be computed accurately.

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\[
[K_a][U_a]=[F]
\]  \hspace{1cm} (5)

Assuming that measured data are noise free and original structural parameters are known. Following paragraph, demonstrates the above approach with the help of Tower Truss [Sanayei and Onipede(1991)], using distributed stiffness parameter, finite element model.

3.2. Numerical examples on tower truss structure

The Structural configuration is same shown in Fig. 3. Adopting the same material constants as mentioned above. A single concentrated load is applied at each joint. The deflections at each joint are measured. The force matrix \([F]\) is found out and corresponding displacement matrix \([U]\) is measured. Existence of damage is predicted comparing the original undamaged deflection with damaged displacements. Figs. 4 to 7 show various types of displacement DOF and force DOF for different cases.
Fig. 4. Tower truss structure: (Case 1)

Fig. 5. Tower truss structure: (Case 2)
3.3 Results

Considering the various multiple damage member combinations, the existence of damage is detected. Different combinations of force DOF and displacement DOF used are given in Tab.1. for various cases (Figs. 4-7).

Fig. 6. Tower truss structure: (Case 3)

Fig. 7. Tower truss structure: (Case 4)
Table 1 Noise-free data – tower truss structure.

<table>
<thead>
<tr>
<th>Case</th>
<th>Force degree of freedom</th>
<th>Displacement degree of freedom</th>
<th>Damaged member considered</th>
<th>Damaged existence identification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1-8</td>
<td>1-8</td>
<td>All</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>5-8</td>
<td>5-8</td>
<td>All</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>1-4</td>
<td>1-4</td>
<td>1,2,3,4,5</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>1-4</td>
<td>5-8</td>
<td>1,2,3,4,5</td>
<td>Yes</td>
</tr>
</tbody>
</table>

With reference to the above table, a case 3 is explained here. A single concentrated force was applied at the joint 1 along the 1 force DOF direction and all the displacements of joints from joint 1 and 2 is measured, along their displacement degree of freedom 1 to 4. The same force is applied but with a change in the force location i.e. at 2, 3 and 4 force DOF respectively. Corresponding to a force position, all the displacements are measured at the displacement DOF from 1 to 4. The member numbers for this damage case prediction are given in the next column.

The measured displacement response was compared to the actual undamaged displacement response of the structure. On comparison, it was found that the measured displacement response was different from the actual undamaged displacement at some locations. By comparing there displacement matrix it could be easily concluded that the structure is having existence of damage and damage existence is identified. It is to be noted that even if the number of the measurement increased at other locations and it gives error in the measurements. It is not going to produce any change in the correct sensor locations. Because the row-echelon form is able to predict the stiffness parameter s even with small sets of measured data. As shown for case 3 and case 4 of the Tab. 1. Hence, the row-echelon form is justified with only few measurements.

4. Conclusion

The finite element method is revisited and row-echelon form of matrix is used for damage detection using force and displacement data. The noise free structural static response data (deflection) was used for the prediction of existence of damage. Row-echelon form has an advantage of partitioning matrix into subgroup for smaller portion of the structural component monitoring. Even with the few sets of load displacement combination of measurements the existence of damage were predicted. The different loading combinations were used. The existence of damage was for with few measured data.

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References


