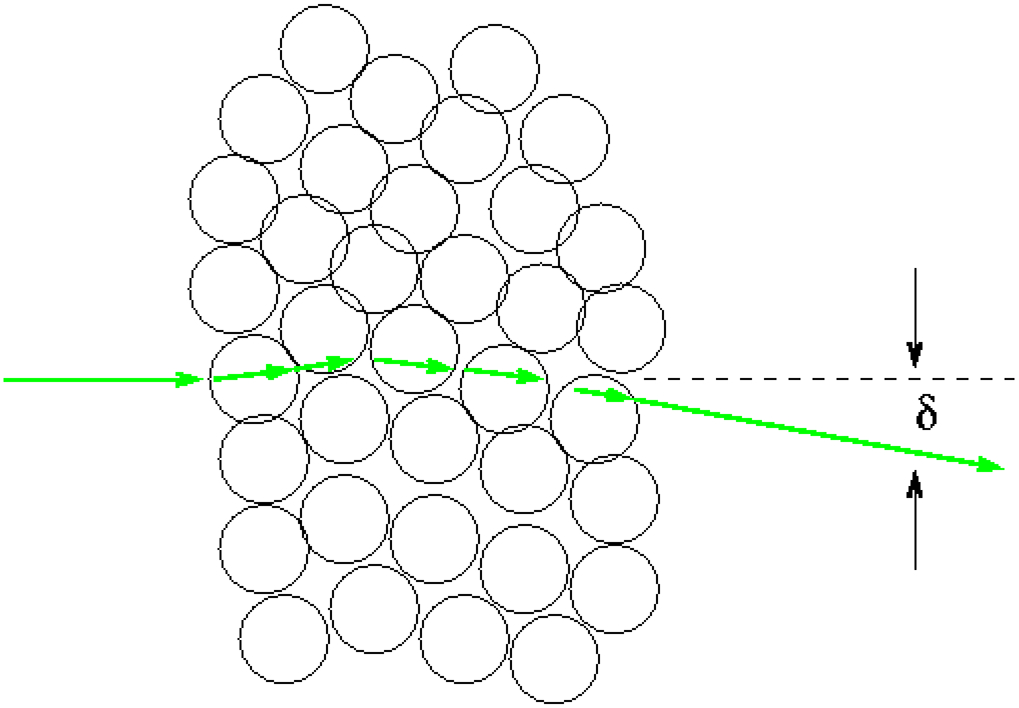
Rutherford and the discovery of the nucleus

So, if the positive charge within an atom is spread out evenly throughout its entire volume, as it Thomson's **plum-pudding model**, one can calculate the typical angle by which an alpha particle might be deflected in an interaction with a single atom:

theta = 0.01 degree

As an alpha particle moves through a thin section of foil, it encounters many atoms. Each atom gives it a small deflection in some random direction. After it comes out of the foil, the alpha particle has some overall deflection **delta** which is the sum of all the small deflections.

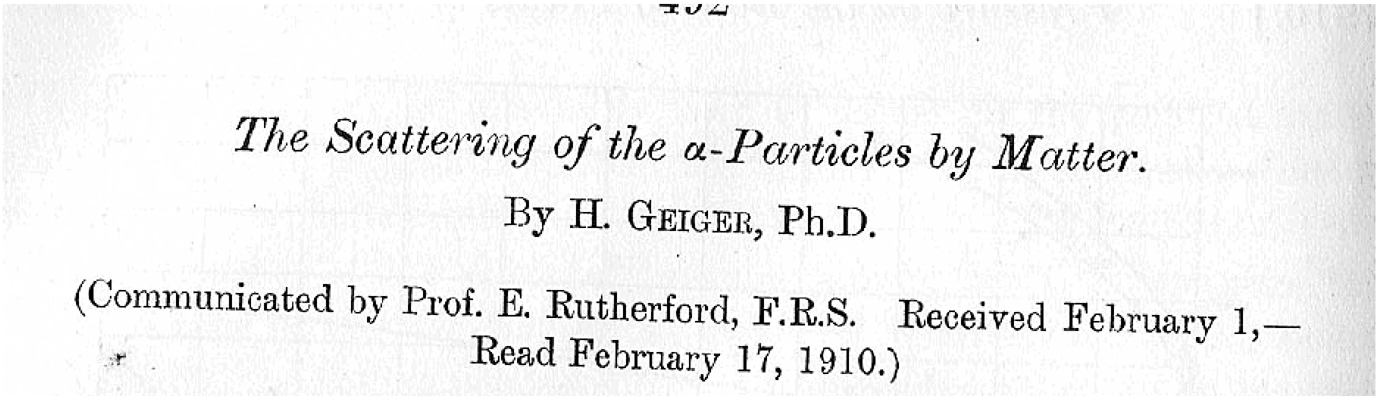


In order to bend by a large angle, say, 90 degrees, all the small deflections would have to be in the same direction: all "to the left", for example. That's not very likely. If there's a 50% chance that each deflection goes "to the left" versus "to the right", what are the chances that consecutive deflections add up to 90 degrees?

Q: How many consecutive deflections in a row would it take to add up to delta=90 degrees overall?

Q: What are the odds that this happens to one
particular alpha particle?

In the early twentieth century, English physicist Ernest Rutherford ran a lab in which experiments of all kinds were performed. One of his assistants, Hans Geiger, investigated scattering of alpha particles by thin films of different metals. In this paper from 1910,



The Scattering of the α-Particles by Matter.

By H. Geiger, Ph.D.

(Communicated by Prof. E. Rutherford, F.R.S. Received February 1,—
Read February 17, 1910.)

Geiger reports on his work. He set up an evacuated glass tube with a radioactive material (radium) coating the walls of the tube at one end, and a flourescent screen at the other end:

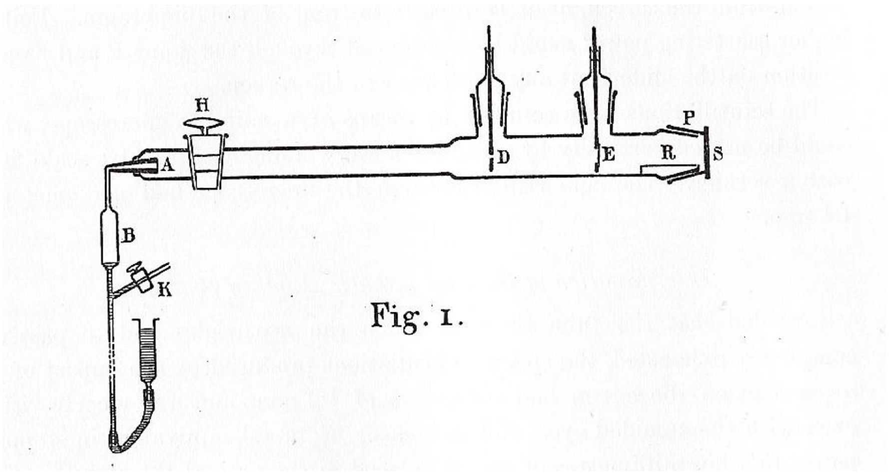
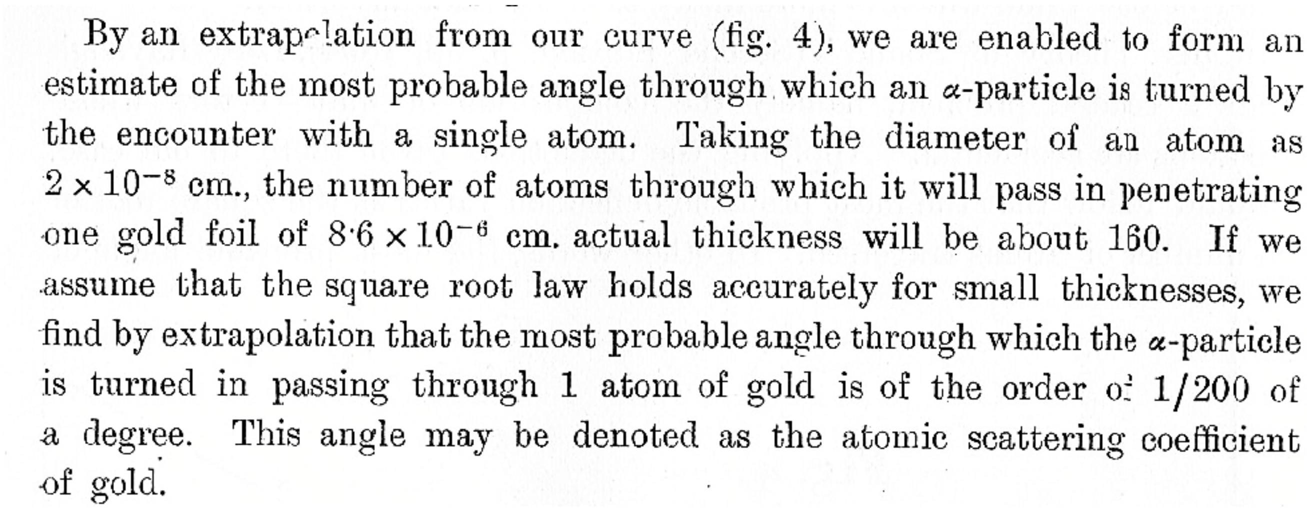


Fig. 1.

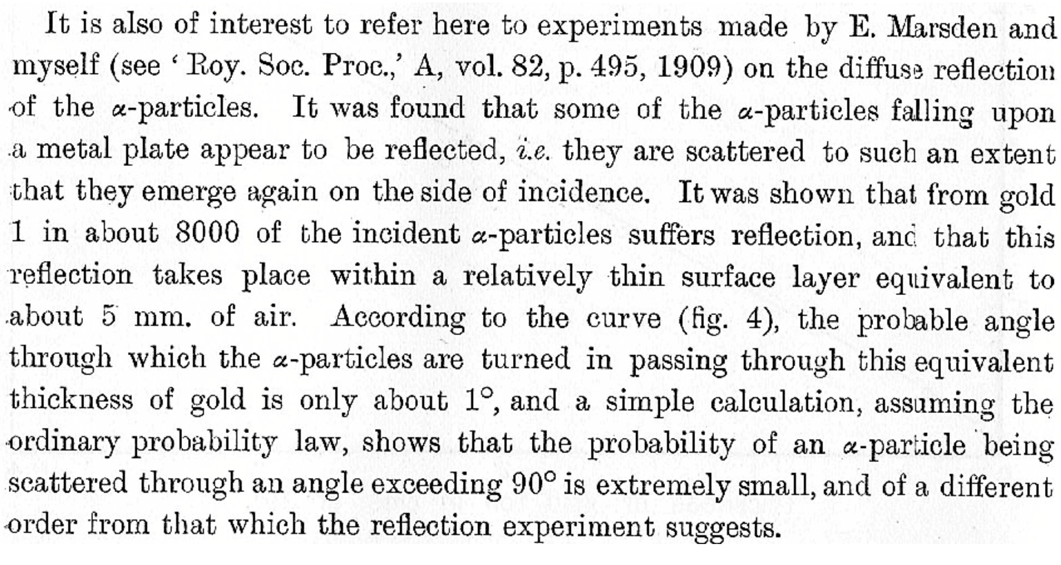
The conical section of glass, **A**, was coated with radium. After about fifteen minutes, the most active radioactive materials disappear, leaving only a single isotope which emitted alpha particles. These flew down the length of the tube, through a small diaphragm **D**, then struck a foil at **E**. At the very end of the tube was a flourescent screen **S**, which would light up with a tiny flash whenever an alpha particle struck it. Geiger would very carefully count the number of flashes at various points on the screen, and derive the angle through which the alpha particles had been deflected by the screen.

Now, Geiger used the Thomson model to interpret his results. He found a typical scattering angle which was indeed small:



> By an extrapolation from our curve (fig. 4), we are enabled to form an estimate of the most probable angle through which an α-particle is turned by the encounter with a single atom. Taking the diameter of an atom as 2×10^{-8} cm., the number of atoms through which it will pass in penetrating one gold foil of $8{\cdot}6 \times 10^{-6}$ cm. actual thickness will be about 160. If we assume that the square root law holds accurately for small thicknesses, we find by extrapolation that the most probable angle through which the α-particle is turned in passing through 1 atom of gold is of the order of 1/200 of a degree. This angle may be denoted as the atomic scattering coefficient of gold.

But there was one little surprise: he occasionally saw a flash of light on a screen placed on the OTHER side of the foil, meaning that an alpha particle had bounced BACKWARDS.



> It is also of interest to refer here to experiments made by E. Marsden and myself (see 'Roy. Soc. Proc.,' A, vol. 82, p. 495, 1909) on the diffuse reflection of the α-particles. It was found that some of the α-particles falling upon a metal plate appear to be reflected, *i.e.* they are scattered to such an extent that they emerge again on the side of incidence. It was shown that from gold 1 in about 8000 of the incident α-particles suffers reflection, and that this reflection takes place within a relatively thin surface layer equivalent to about 5 mm. of air. According to the curve (fig. 4), the probable angle through which the α-particles are turned in passing through this equivalent thickness of gold is only about 1°, and a simple calculation, assuming the ordinary probability law, shows that the probability of an α-particle being scattered through an angle exceeding 90° is extremely small, and of a different order from that which the reflection experiment suggests.

He determined that about 1 in 8000 alpha particles bounced backwards. But if the typical scattering angle is just 1/200 of a degree, then it should take a number of consecutive "left-hand" turns **N** given by

$$N = \frac{90 \text{ degrees}}{1/200 \text{ degree}} = 18{,}000$$

If there's a 50% chance of scattering to the left or the right each time the alpha encounters an atom, the probability of 18,000 consecutive left-hand turns is

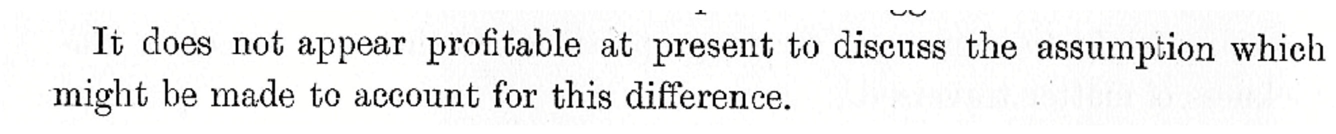
$$\text{prob } P = 2^{-18{,}000} = 10^{-6{,}000}$$

which is a lot smaller than 1 in 8000. A LOT smaller. A REALLY BIG LOT smaller!

In the words of Rutherford,

It was quite the most incredible event that ever happened to me in my life. It was as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.

So what was going on? The Thomson model can't explain it. In the old-fashioned tradition of being cautious (which doesn't exist much anymore -- it doesn't get press), Geiger writes:



It does not appear profitable at present to discuss the assumption which might be made to account for this difference.

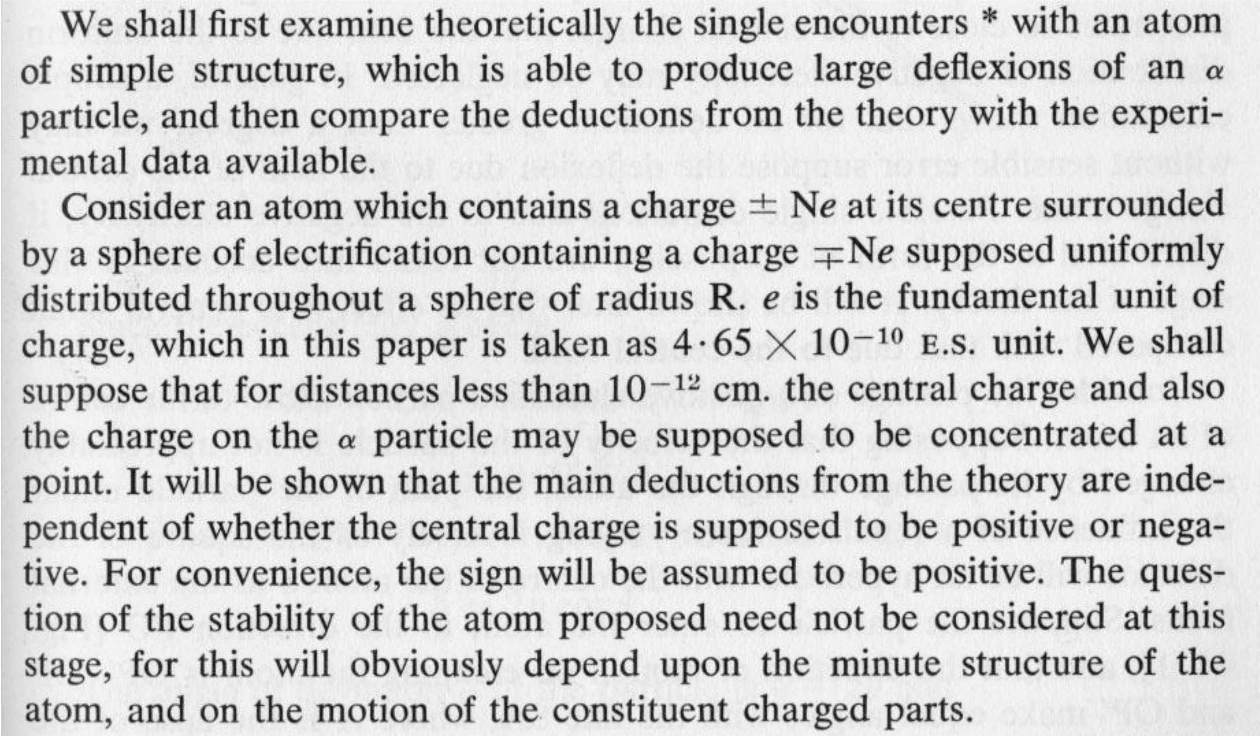
But he (and Rutherford) did have an idea ... one that they published three years later, after an exhaustive series of similar experiments.

Rutherford's model: the positive nucleus

Rutherford realized that a bunch of weak scattering events would never lead to the observed frequency of angles greater than 90 degrees. For one thing, the thin foils he and his colleagues were using were only several hundred atoms thick; there just wasn't time for an alpha particle to encounter the thousands of atoms needed to bend its path by large angles.

Perhaps each alpha underwent just one scattering event -- but one which could sometimes be very strong.

The experiments showed several relationships between the number of alpha particles scattered at some angle **theta** and other quantities: the type of material in the foil, its thickness, the kinetic energy of the incoming alpha particles, etc. Rutherford worked out a model which could explain all the correlations.



We shall first examine theoretically the single encounters * with an atom of simple structure, which is able to produce large deflexions of an α particle, and then compare the deductions from the theory with the experimental data available.

Consider an atom which contains a charge $\pm Ne$ at its centre surrounded by a sphere of electrification containing a charge $\mp Ne$ supposed uniformly distributed throughout a sphere of radius R. e is the fundamental unit of charge, which in this paper is taken as $4 \cdot 65 \times 10^{-10}$ E.S. unit. We shall suppose that for distances less than 10^{-12} cm. the central charge and also the charge on the α particle may be supposed to be concentrated at a point. It will be shown that the main deductions from the theory are independent of whether the central charge is supposed to be positive or negative. For convenience, the sign will be assumed to be positive. The question of the stability of the atom proposed need not be considered at this stage, for this will obviously depend upon the minute structure of the atom, and on the motion of the constituent charged parts.

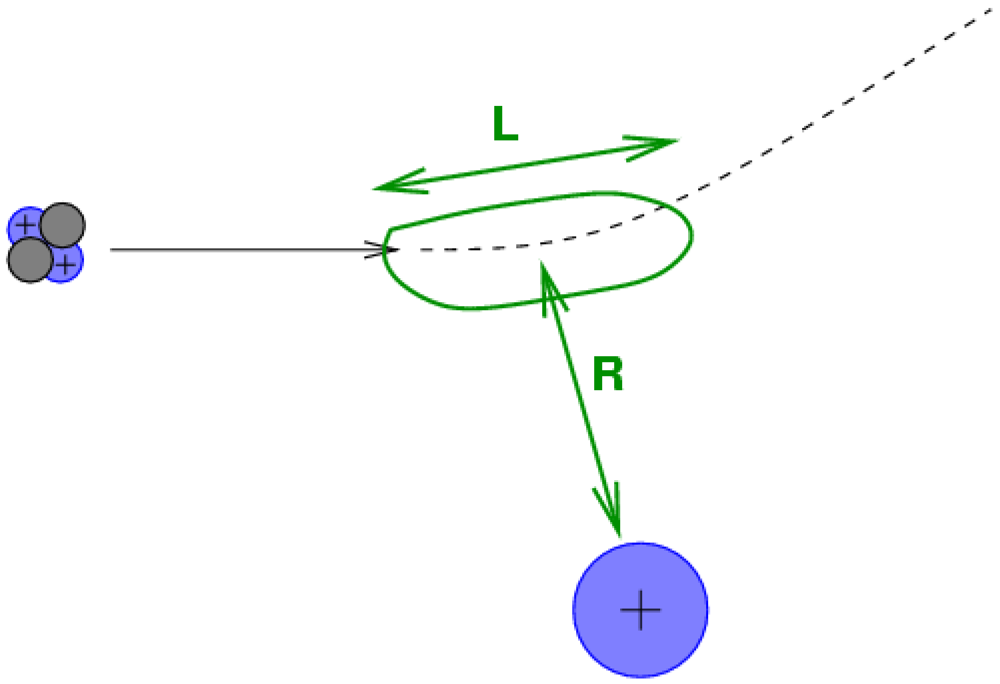
From "The Scattering of alpha and beta Particles by Matter and the Structure of the Atom," *Philosophical Magazine*, vol. 21, p. 669 (1911)

In this new model, all the positive charge of the atom was concentrated in a tiny point at the center; around it was nearly empty space, inhabited by the electrons alone. As Rutherford put it, the atom wasn't a plum pudding; it was more like

... like a few flies in a cathedral ...

The crucial point was the very small size of the "nucleus": since all the positive charge was concentrated in so small a volume, the projectile could OCCASIONALLY come very, very close to it; so close that the electric force would be large enough to push the projectile by a very large angle, maybe even backwards.

Let's use the impulse approximation again to get a feeling for what's happening.



If the alpha particle approaches the nucleus to a minimum distance **R**, then the impulse imparted to the alpha by the electric force is roughly the force at closest approach multipled by the duration of the passage:

$$
\begin{aligned}
\text{impulse} &\sim \quad F \qquad\qquad t \\
&\sim \frac{k(2e)(Ne)}{R^2} \quad \frac{L}{v} \\
&\sim \frac{k(2e)(Ne)}{R^2} \quad \frac{2R}{v} \\
&\propto \quad \frac{1}{R^2} \qquad\quad R
\end{aligned}
$$

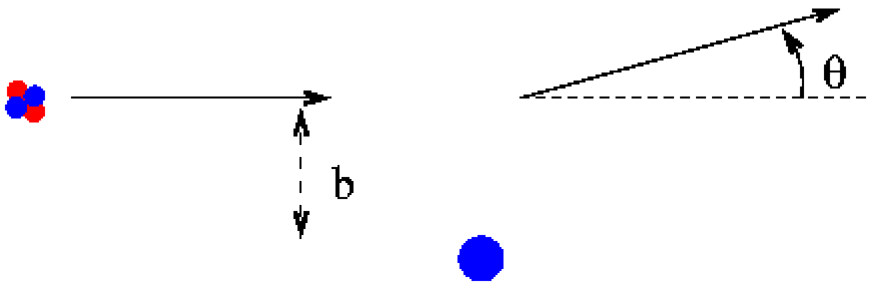
Q: What happens to this impulse as the distance R decreases?

Q: What is the rough size of an atom?

Q: What is the rough size of a nucleus?

Q: How much stronger is the impulse if the positive charge is compressed within the nucleus, rather than being spread throughout the entire volume of an atom?

Using his model, Rutherford derived two formulae which described the behavior of alpha particles as a function of the other variables in the experiment. First, he could determine the impact parameter **b** of an charged particle which was scattered by an angle **theta**:



Given the Coulomb force constant **ke**, the charge **z** (in electron units) on the projectile, the charge **Z** (in electron units) on the atom, and the kinetic energy **K** of the projectile, Rutherford's model yields:

$$b = \frac{k_e z Z e^2}{2K} \frac{1}{\tan(\frac{1}{2}\theta)}$$

One can use this formula to derive equation 6.15 in your textbook. That derived relationship concerning the fraction of particles scattered by more or less than a particular angle will come in handy on this week's homework assignment.

If one measures the fraction of particles which are scattered by various angles as they pass through a foil, one can work out the fraction of the cross-section of an atom which is occupied by the nucleus. In other words, **one can measure the size of the nucleus!**

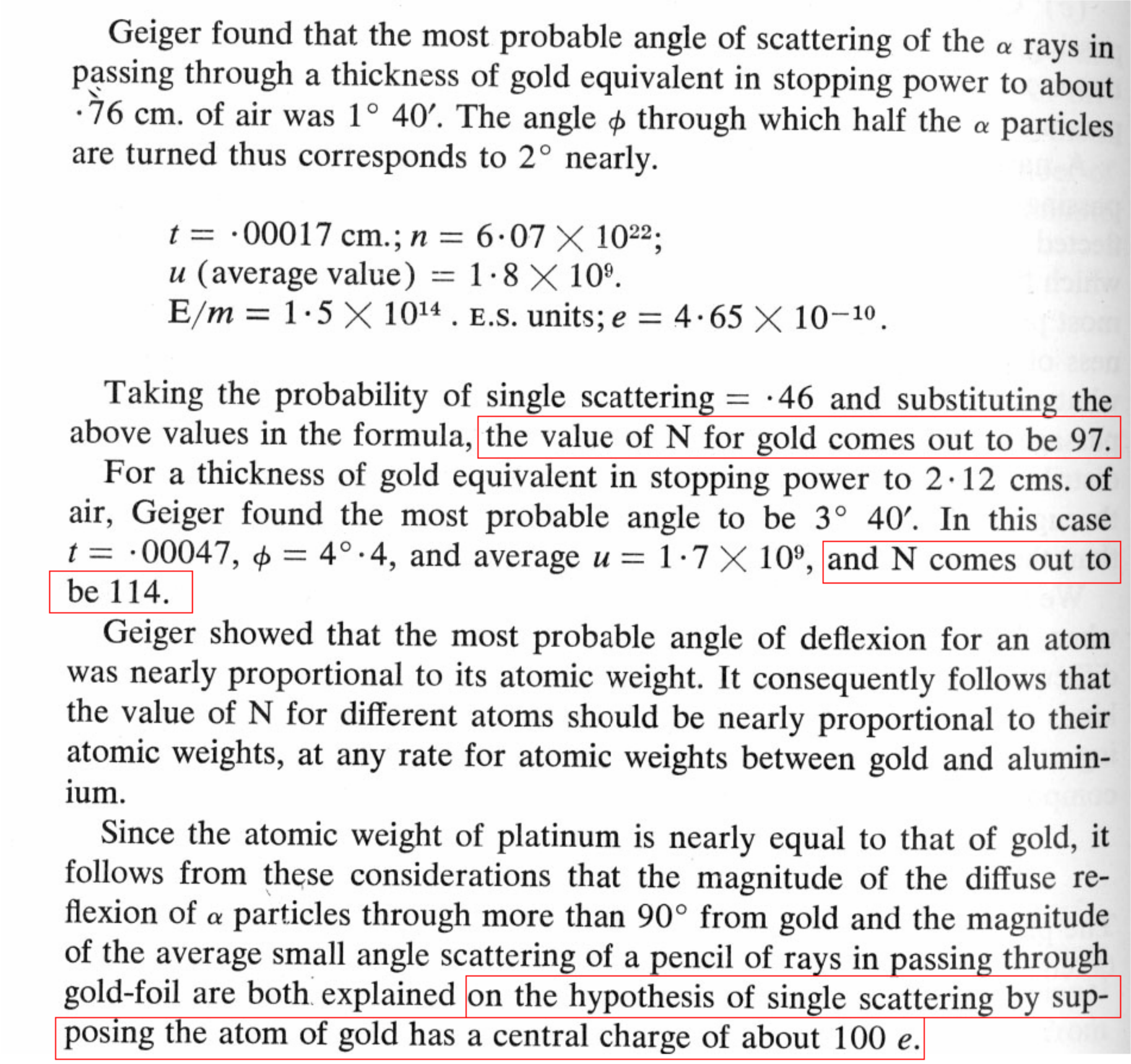
Rutherford's model also makes a series of predictions between the fraction of particles scattered at a particular angle and a number of quantities involved in the experiment:

- composition of foil
- thickness of foil
- kinetic energy of projectile particles
- charge on the projectile particles

Rutherford was able to put all this information into a single complicated formula:

$$N(\theta) = \frac{nt}{4r^2}\left(\frac{zZ}{2K}\right)^2\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\frac{1}{\sin^4(\frac{1}{2}\theta)}$$

Using this formula, and the experimental results of Geiger and Marsden, Rutherford was able to figure out the approximate electric charge of the nucleus of a gold atom:



Geiger found that the most probable angle of scattering of the α rays in passing through a thickness of gold equivalent in stopping power to about ·76 cm. of air was 1° 40′. The angle φ through which half the α particles are turned thus corresponds to 2° nearly.

$t =$ ·00017 cm.; $n = 6{\cdot}07 \times 10^{22}$;
u (average value) $= 1{\cdot}8 \times 10^{9}$.
$E/m = 1{\cdot}5 \times 10^{14}$. E.S. units; $e = 4{\cdot}65 \times 10^{-10}$.

Taking the probability of single scattering = ·46 and substituting the above values in the formula, the value of N for gold comes out to be 97.

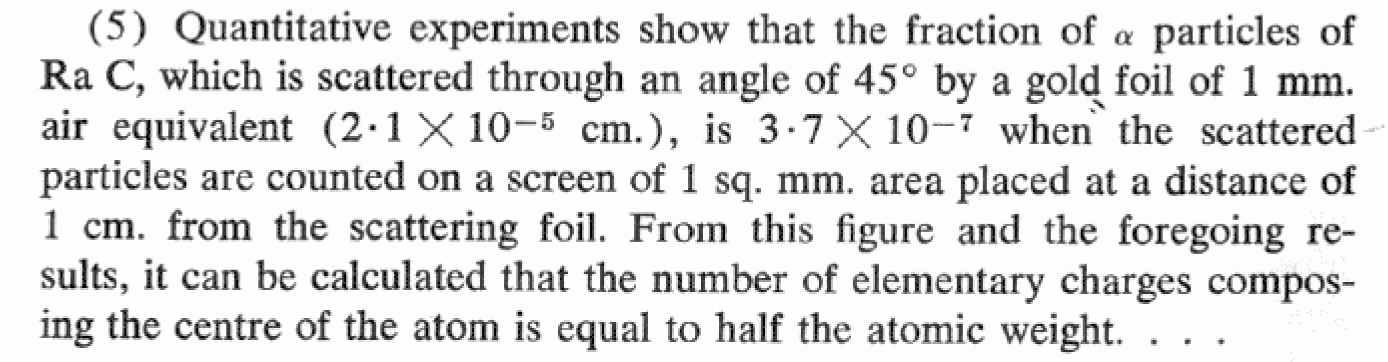
For a thickness of gold equivalent in stopping power to 2·12 cms. of air, Geiger found the most probable angle to be 3° 40′. In this case $t =$ ·00047, $\phi = 4°{\cdot}4$, and average $u = 1{\cdot}7 \times 10^{9}$, and N comes out to be 114.

Geiger showed that the most probable angle of deflexion for an atom was nearly proportional to its atomic weight. It consequently follows that the value of N for different atoms should be nearly proportional to their atomic weights, at any rate for atomic weights between gold and aluminium.

Since the atomic weight of platinum is nearly equal to that of gold, it follows from these considerations that the magnitude of the diffuse reflexion of α particles through more than 90° from gold and the magnitude of the average small angle scattering of a pencil of rays in passing through gold-foil are both explained on the hypothesis of single scattering by supposing the atom of gold has a central charge of about 100 *e*.

What's the currently accepted value for the electric charge in a gold nucleus?

One of the somewhat surprising results of the experiments was that the variations in angle didn't run exactly as one might expect with elements of increasing atomic weight. Naively, one might expect that an element with an atomic weight of **N** might have **N** positive charges at its center. However, as Geiger and Marsden write in the conclusion to their paper **The Laws of Deflexion of alpha Particles through Large Angles**, *Philosophical Magazine*, vol 25, p. 604 (1913):



> (5) Quantitative experiments show that the fraction of α particles of Ra C, which is scattered through an angle of 45° by a gold foil of 1 mm. air equivalent ($2\cdot 1 \times 10^{-5}$ cm.), is $3\cdot 7 \times 10^{-7}$ when the scattered particles are counted on a screen of 1 sq. mm. area placed at a distance of 1 cm. from the scattering foil. From this figure and the foregoing results, it can be calculated that the number of elementary charges composing the centre of the atom is equal to half the atomic weight. . . .

Can you explain why the number of positive charges **N** in a nucleus was only about HALF the atomic weight?

Summary of the scattering results, as of 1913

1. The positive charge and almost all the mass of an atom is contained in a very compact nucleus, the size of which is roughly 10^(-15) m.
2. One can estimate the electric charge within an atomic nucleus roughly.
3. The positive charge of the nucleus (in multiples of the charge of an electron) is roughly equal to half the atomic weight (in multiples of hydrogen's mass).

Quite an impressive amount of information, no? As an analogy, imagine trying to discern the internal structure of an automobile from a distance of several hundred yards by shooting a high-powered rifle at it.

Source: http://spiff.rit.edu/classes/phys314/lectures/rutherford/rutherford.html