

# Formulation of Equivalent Steel Section for Partially Encased Composite Column under Concentric Gravity Loading

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**Abstract** - Partially encased composite column (PEC) consists of thin walled welded H- shaped steel section with transverse links provided at regular intervals between the flanges to inhibit the occurrence of local buckling in the thin flange plates. The space between the flanges and the web plate are filled up with concrete. Extensive experimental investigations have been conducted by several research groups to understand the behavior of this relatively new composite column under both concentric and eccentric loading conditions along with sophisticated non-linear finite element analysis. But the separation between concrete and steel initiates the unstable condition in the finite element analysis near the ultimate point when flange plate buckles. To avoid the expensive and cumbersome modeling of the behavior at the interface of two dissimilar materials, an attempt has been made in this study to replace the composite section of PEC column with an equivalent steel section which can easily be incorporated in commercially available finite element softwares.

**Key words** - column: composite; concrete: encased; equivalent; steel.

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## I. INTRODUCTION

Steel-concrete composite columns can substantially improve the behavior and cost efficiency of steel-only columns used in the construction of mid-rise and high-rise buildings. A welded H-shaped steel section figures the partially encased composite (PEC) section with concrete infill between the flanges (Fig. 1). In Europe, in the early 1980s, PEC columns and beams were introduced using standard-sized rolled steel sections. In 1996, the Canam Group in North America proposed a PEC column section constructed from a thin-walled built-up steel shape with transverse links provided at regular intervals to restrain local buckling as shown in Fig.1. Extensive experimental research has been performed in Ecole Polytechnique de Montréal [1-3] on small-scale and large-scale PEC column specimens under various conditions of loading. The influences of high performance materials on the behavior of these columns have also been investigated experimental by Prickett and Driver [4] at the University of Alberta. The results of these experimental investigations indicated that the behavior of this composite column is significantly affected by the local instability of the thin steel flanges. Begum et al. [5] were able to overcome these challenges in the finite element model through the implementation of a dynamic explicit formulation along with a damage plasticity model for concrete and a contact pair algorithm at the steel-concrete interface. The developed model was applied successfully to

reproduce the behavior of 34 PEC columns from five experimental programs. However, despite of the accuracy, the composite finite element model developed by Begum et al. [5] is very sophisticated due to the presence of two dissimilar materials. Moreover, modeling of the interfacial behavior between steel and concrete requires extensive calculations as well as skilled and experienced users. Due to these complexities most of the structural analysis and design software do not handle such composite members. In this paper an attempt has been made to develop a fictitious steel section of the partially encased composite steel section. This equivalent steel section has added a new dimension to the analysis of partially encased composite column renovating the numerical procedure.

## II. OBJECTIVES AND SCOPES

The main objective of this study is to formulate the methodology for the development of fictitious steel section for partially encased composite column built up with thin steel plates. The methodology is then used to simulate ten test PEC test columns from the published literature with variety of geometric properties subjected to concentric gravity loading only. The formulation of the fictitious steel section is based on the equivalence criteria of the basic geometric properties of the composite column. Linear elastic material behavior is assumed for steel as well as for concrete in the composite section.

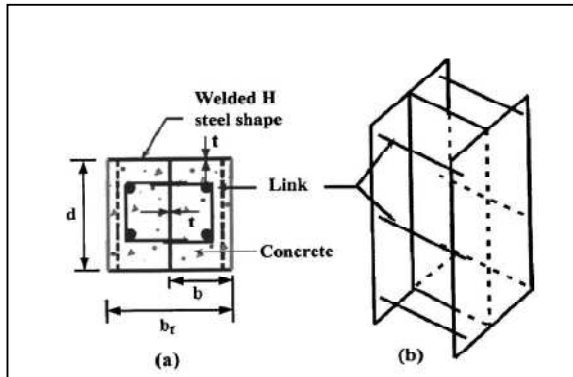


Fig. 1 : Partially Encased Composite Column with Thin-Walled Built-Up Steel Section, (a) Column Cross-Section and (b) 3D view of the Steel Configuration

### III. REFERENCE TEST COLUMNS

Ten test PEC columns from the published literature are selected for current study. The lists of these specimens, along with their properties, are given in Table 1. Fig. 2 shows the cross-section and elevation of a typical test column. Specimens C-2 to C-7 were tested during the initial phase of the research program by Tremblay et al. [1] to study the behavior under concentric gravity loading which had square cross-sections of 300 mm × 300 mm and 450 mm × 450 mm, and a length equal to 5d, where d is the depth of the cross-section. Specimens C-8 to C-11, tested by Chicoine et al. [2] under axial compression, were larger in their cross-sectional dimensions (600 mm×600 mm) as compared to the previous test specimens.

All the test columns were fabricated from CSA-G40.21-350W grade steel plate. Normal strength concrete (nominally 30 MPa) was used in the test region of these columns. To strengthen the end regions of these test specimens, high strength concrete of 60 MPa

nominal strength was used along with the closer link spacing provided in these zones.

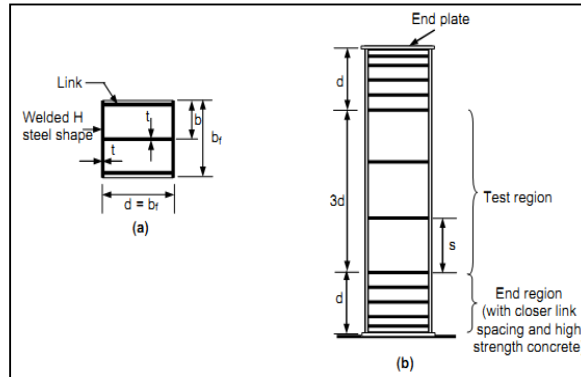


Fig. 2 : Typical PEC Test Column, (a) Cross-section, (b) Elevation

### IV. FORMULATION OF EQUIVALENT STEEL SECTION

The fictitious steel cross-section consists of the actual steel cross-section and two additional pairs of plates, one perpendicular to the web at mid-height and one perpendicular to the flanges at mid-width as shown in Fig.3. The cross-sections of the fictitious and the actual column are equivalent in that they have the same:

1. Resistance in compression, and
2. Bending stiffness about the two principal centroidal axes.

In order to produce the equivalent fictitious steel section, the dimensions of the two pairs of steel plates added to the initial cross-section is to be determined from the algebraic expressions of the three equivalence criteria: compression resistance and bending stiffness about the two principal axes.

TABLE 1 : PROPERTIES OF REFERENCE TEST SPECIMENS

Reference	Specimen	Plate size $b_f \times d \times t$	Link spacing $s$	Link diameter $\Phi$	Length $L$	Compressive Strength of concrete	Yield strength of plates
		(mm)	(mm)	(mm)	(mm)	(MPa)	(MPa)
Tremblay et al. (1998)	C-2	450x450x9.70	225	12.7	2250	32.7	370
	C-3	450x450x9.70	337.5	12.7	2250	32.4	370
	C-4	450x450x9.70	450	12.7	2250	31.9	370
	C-5	450x450x9.70	225	22.2	2250	34.3	370
	C-6	450x450x6.35	337.5	12.7	2250	33.1	374
	C-7	300x300x6.35	300	12.7	1500	31.9	374

Chicoine et al. (2002)	C-8	600x600x12.88	600	15.9	3000	34.2	360
	C-9	600x600x12.91	600	15.9	3000	34.2	360
	C-10	600x600x12.81	300	15.9	3000	34.2	360
	C-11	600x600x9.70	600	15.9	3000	34.2	345

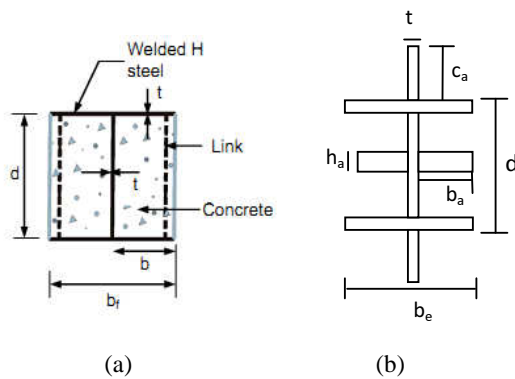


Fig. 3 : Equivalent steel section for PEC column, (a) Actual cross-section, (b) Equivalent steel section

#### A. Equivalence in compression resistance

The compression resistance of a composite steel-concrete cross section comprises of the plastic resistance of the steel cross-section, steel links and the concrete. The total area of the fictitious cross-section represents the area combining area of the initial steel cross-section and the additional plates.

$$P_{actual} = A_{se}f_s + A_c f_c$$

$$P_{fictit} = (A_{se} + A_a)f_s \quad (1)$$

Here,  $A_{se}$  = effective area of the steel shape including the I section and the links

$A_a$  = total steel area of the additional steel plates in fictitious section

$f_s$  = design stress of steel =  $f_y$

$f_c$  = design stress of concrete

The effective area of a non-compact steel section can be defined as,

$$A_{se} = (d - 2t + 2b_e)t \quad (2)$$

Here, overall depth of the cross section

Thickness of the steel plates

Total effective width of the flange

$$\text{Now, } b_e = \frac{b_f}{(1 + \lambda_p^{2n})^{1/n}} \leq b_f$$

$$\lambda_p = \frac{b}{t} \sqrt{\frac{12(1 - \nu_s^2)F_y}{\pi^2 E_s k}}$$

$$k = 0.9/(s/b_f) + .2(s/b_f)2 + .75 \quad (.5 \leq s/b_f \leq 1)$$

Here,  $b_f$  = full width of a flange plate

$\lambda_p$  = Slenderness parameter

$E_s$  = Elastic modulus of steel

$\nu_s$  = Poisson's ratio of steel

$n = 1.5$  as proposed by Chicoine et al. [3]

Considering the condition of equivalence in compression resistance and the relation in (1), it is obtained

$$A_a = A_c \frac{f_c}{f_s} = A_c m_{cd} \quad (3)$$

Here,  $m_{cd}$  = design-stress ratio for concrete-to-steel

The total area of the composite cross-section comprises the equivalent steel cross section  $A_{se}$  and concrete section  $A_c$ .

$$b_f d \equiv A_{se} + A_c \quad (4)$$

Dividing Eq. (2) by  $b_f d$  and introducing the non dimensional parameters  $\rho_{se}$  and a positive constant  $q_x^2$ ,

$$A_a / b_f d = m_{cd} \rho_{se}^n = q_x^2 \quad (5)$$

The total cross-sectional area of the four additional plates in the fictitious section as shown in Fig.3 is,

$$A_a = 2b_a \times h_a + 2c_a \times t \quad (6)$$

Which after normalization by  $b_f d$  and introduction of normalized plate dimensions  $\beta$ ,  $\eta$ ,  $\chi$  and  $\gamma$  becomes:

$$A_a/b_f d = (2\beta)\eta + (2\chi)\gamma \quad (7)$$

$$\rho_{se} = A_{se}/b_f d$$

Since equation (4) and (6) are identical in left hand sides, the outcome is:

$$q_x^2 = (2\beta)\eta + (2\chi)\gamma \quad (8)$$

Here the unknowns are  $\beta$ ,  $\eta$  and  $\chi$  on the left side and the combined geometric and material attributes of the actual cross-section enter through the constants  $\gamma$  and  $q_x^2$ .

#### B. Equivalence in stiffness about the major axis

The flexural stiffness about the major axis y-y of the actual composite steel-concrete cross-section and its fictitious purely steel counterpart are given by:

$$(EI)_{y,actual} = E_{se}I_{se,y} + E_{ce}I_{c,y} \\ (EI)_{y,fictit} = E_{se}I_{se,y} + E_{se}I_{a,y} \quad (9)$$

Here,  $E_{se}$  and  $E_{ce}$  are the effective elastic modulus of steel and concrete respectively.  $I_{se,y}$ ,  $I_{c,y}$  and  $I_{a,y}$  are the moment of inertia about minor axis for actual steel section, concrete and additional steel plates respectively.

To enforce the condition of equivalence in major-axis stiffness, solving for  $I_{a,y}$  and introducing the concrete-to-steel ratio of elasticity moduli,

$$I_{a,y} = \left( \frac{E_{ce}}{E_{se}} \right) \times I_{c,y} = m_e I_{c,y} \quad (10)$$

In reference to Fig.1(a)

$$b_f d^3 / 12 \equiv I_{se,y} + I_{c,y}$$

$$I_{se,y} \equiv A_{se} r_{se,y}^2$$

Equation (9) may be written as:

$$I_{a,y} = m_e \left( b_f d^3 / 12 - A_{se} r_{se,y}^2 \right) \quad (11)$$

After normalization of (11) by dividing it with  $b_f d^3 / 12$  and using  $q_y^2$  as a positive constant to sum up the effects of the geometry and material properties of the actual cross section, the final equation is,

$$\left( 12I_a / b_f d^3 \right) = m_e (1 - 3\rho_{se} \times \lambda_{se,y}^2) = q_y^2 \quad (12)$$

Here,  $\lambda_{se,y} = 2 r_{ay} / d$

The additional major-axis flexural stiffness of the fictitious cross-section as a function of the dimensions of the additional plates is given by:

$$I_a = \left( 2 \frac{b_a h_a^3}{12} \right) + \left( t / 12 \right) \times \{ (2c_a + d)^3 - d^3 \}$$

This, after normalization with  $(b_f d^3 / 12)$  and introduction of the non-dimensional geometric properties  $\beta$ ,  $\eta$ ,  $\chi$ , and  $\gamma$ , yields:

$$12I_{a,y} / b_f d^3 = (2\beta)\eta^3 + \gamma \{ (2\chi + 1)^3 - 1 \} \quad (13)$$

Since (12) and (13) have identical left-hand sides,

$$(2\beta)\eta^3 + \gamma \{ (2\chi + 1)^3 - 1 \} = q_y^2 \quad (14)$$

Here the unknowns are  $\beta$ ,  $\chi$  on the left side and the combined geometric and material attributes of the actual cross-section enter through the constants  $\gamma$  and  $q_y^2$ .

#### C. Equivalence in stiffness about minor axis

The treatment of major axis stiffness equivalence presented in the previous subsection is repeated here in shorthand for the minor-axis case.

First, minor-axis counterpart of (11):

$$I_{a,z} = m_e \left( \frac{b_f^3 d}{12} - I_{se,z} \right) \\ = m_e \left( \frac{b_f^3 d}{12} - A_{se} r_{se,z}^2 \right) \quad (15)$$

After normalization of (15) by dividing it with  $b_f d^3 / 12$  and using  $q_z^2$  as a positive constant the obtained equation is

$$\left( 12I_{a,z} / b_f^3 d \right) = m_e (1 - 3\rho_{se} \times \lambda_{se,z}^2) = q_z^2 \quad (16)$$

Here,  $\lambda_{se,z} = 2 r_{az} / d$

$$\rho_{se} = A_{se} / b_f d$$

Similarly for the additional elements of the fictitious steel cross-section, one obtains the counterpart of (13),

$$12I_{a,z}/b^3d = \{(2\beta + \gamma)^3 - \gamma^3\}\eta + (2\chi)\gamma^3 \quad (17)$$

Since (16) and (17) have identical left-hand sides, the outcome is

$$\{(2\beta + \gamma)^3 - \gamma^3\}\eta + (2\chi)\gamma^3 = q_z^2 \quad (18)$$

Here the unknowns are  $\beta$ ,  $\chi$  on the left side and the combined geometric and material attributes of the actual cross-section enter through the constants  $\gamma$  and  $q_z^2$ .

In order to transform the actual section in to fictitious section, three non-linear (8), (14) and (18) were developed. Three non-dimensional parameters,  $q_x^2$ ,  $q_y^2$  and  $q_z^2$  describe the additional compression resistance major-axis stiffness, and minor-axis stiffness, respectively, due to concrete and links. These equations must be solved for the three non-dimensional unknowns'  $\beta$ ,  $\chi$  and  $\eta$  to determine the dimensions of the additional steel plate's  $b_a$ ,  $h_a$ , and  $c_a$  in the fictitious cross-section.

## V. COMPARISON OF FICTITIOUS SECTION WITH ACTUAL SECTION

The methodology described in the previous section is implemented to compute the equivalent steel sections of the reference test PEC columns mentioned in Table I. The dimensions of the additional steel plates representing the concrete part of the composite section are provided in Table II. The geometric properties of the equivalent steel sections for columns C-2 to C-11 is calculated and compared to their actual composite cross-section. Table III presents the comparison between the cross-sectional area and moment of inertias of the composite and equivalent steel section of the column. The results show that the cross-sectional area and the moment of inertia about the major axis for equivalent steel section are in excellent agreement with the actual composite section. The moment of inertia about minor axis of the steel section is found to be around 4% lower than that of the composite section. This difference is within acceptable range.

Table II : Development of Equivalent Steel Section for Partially Encased Composite Column

Specimen	Composite steel section					Equivalent steel section				
	<i>bf</i>	<i>d</i>	<i>t</i>	<i>s</i>	$\phi$	<i>be</i>	<i>ba</i>	<i>ha</i>	<i>ca</i>	<i>t</i>
C-2	450	450	9.7	225	12.7	368.08	304.90	21.46	196.36	9.7
C-3	450	450	9.7	337.5	12.7	371.48	304.91	21.45	196.36	9.7
C-4	450	450	9.7	450	12.8	375.89	309.98	20.42	196.36	9.7
C-5	450	450	9.7	225	22.2	368.08	295.48	23.54	196.36	9.7
C-6	450	450	6.35	337.5	12.7	265.46	303.64	23.19	259.36	6.35
C-7	300	300	6.35	300	12.7	247.74	223.58	12.18	142.27	6.35
C-8	600	600	12.88	600	15.9	503.43	382.01	33.51	258.57	12.88
C-9	600	600	12.91	600	15.9	503.97	382.00	33.50	258.15	12.91
C-10	600	600	12.81	300	15.9	491.82	382.00	33.58	259.57	12.81
C-11	600	600	9.7	600	15.9	420.41	382.21	35.09	312.89	9.7

Table III. Comparison between the properties of composite and fictitious section

Specimens	Area			Moment of inertia about of major axis			Moment of inertia about minor axis		
	<i>CPS</i> *10 <sup>3</sup>	<i>EQS</i> *10 <sup>3</sup>	$\frac{A_{CPS}}{A_{EQS}}$	<i>CPS</i> *10 <sup>6</sup>	<i>EQS</i> *10 <sup>6</sup>	$\frac{I_{y,CPS}}{I_{y,EQS}}$	<i>CPS</i> *10 <sup>6</sup>	<i>EQS</i> *10 <sup>6</sup>	$\frac{I_{z,CPS}}{I_{z,EQS}}$
C-2	16.76	16.90	0.99	410.13	410.63	1.00	443.14	425.22	1.04
C-3	16.76	16.89	0.99	410.13	410.63	1.00	443.14	425.06	1.04
C-4	16.35	16.47	0.99	410.13	410.57	1.00	443.14	424.86	1.04
C-5	17.58	17.72	0.99	410.13	410.77	1.00	443.14	425.22	1.04

C-6	17.17	17.38	0.99	432.84	433.47	1.00	455.13	446.56	1.02
C-7	7.20	7.25	0.99	91.40	91.47	1.00	98.61	94.67	1.04
C-8	32.03	32.27	0.99	1264.62	1267.01	1.00	1365.96	1309.64	1.04
C-9	32.02	32.26	0.99	1264.15	1266.55	1.00	1365.71	1309.21	1.04
C-10	32.04	32.30	0.99	1265.69	1268.10	1.00	1366.54	1311.73	1.04
C-11	32.56	32.89	0.99	1314.18	1316.93	1.00	1392.22	1356.36	1.03
MEAN	0.99			1.00					
SD	0.01			0.00			0.00		

## VI. CONCLUSIONS

The composite cross-section of thin walled partially encased composite column was replaced by an equivalent steel section. The formulation of the fictitious steel plates replacing the concrete part of the composite cross-section was presented in the paper. Linear elastic material behavior was considered during the formulation. The local buckling of the thin flange plates were incorporated by considering the effective width of the flange plates. The proposed methodology was used to calculate the equivalent steel section of ten PEC columns with a variety of geometric properties under concentric gravity loading only. The geometric properties of the actual composite section and the equivalent section were compared. The cross-sectional area and moment of inertias for the equivalent steel section were found to be very close to properties of the actual composite section. The proposed methodology can reliably replace the composite section with an equivalent steel section.

## VII. ACKNOWLEDGMENT

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