Design of Gravity Dam by Genetic Algorithms

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Abstract—The design of a gravity dam is performed through an interactive process involving a preliminary layout of the structure followed by a stability and stress analysis. This study presents a method to define the optimal top width of gravity dam with genetic algorithm. To solve the optimization task (minimize the cost of the dam), an optimization routine based on genetic algorithms (GAs) was implemented into an Excel spreadsheet. It was found to perform well and GA parameters were optimized in a parametric study. Using the parameters found in the parametric study, the top width of gravity dam optimization was performed and compared to a gradient-based optimization method (classic method). The accuracy of the results was within close proximity. In optimum dam cross section, the ratio of is dam base to dam height is almost equal to 0.85, and ratio of dam top width to dam height is almost equal to 0.13. The computerized methodology may provide the help for computation of the optimal top width for a wide range of height of a gravity dam.

Keywords—Chromosomes, dam, genetic algorithm, global optimum, preliminary layout, stress analysis, theoretical profile.

I. INTRODUCTION

Basically, gravity dams are solid concrete structures that maintain their stability against design loads from the geometric shape and the mass and strength of the concrete. Generally, they are constructed on a straight axis, but may be slightly curved or angled to accommodate the specific site conditions. Gravity dams typically consist of a nonoverflow section(s) and an overflow section or spillway. The two general concrete construction methods for concrete gravity dams are conventional placed mass concrete and RCC. Dam profiles consist of nonoverflow and overflow sections. The configuration of the nonoverflow section is usually determined by finding the optimum cross section that meets the stability and stress criteria for each of the loading conditions. The design cross section is generally established at the maximum height section and then used along the rest of the nonoverflow dam to provide a smooth profile. The upstream face is generally vertical, but may include a batter/fillet to increase sliding stability or in existing projects provided to meet prior stability criteria for construction requiring the resultant to fall within the middle third of the base. The downstream face will usually be a uniform slope transitioning to a vertical face near the crest. Based on U.S. Army Corps of Engineers [1], the slope will usually be in the range of 0.7H-1V and 0.8H-1V, depending on uplift and the seismic zone, to meet the stability requirements. Two basic loading conditions are used in gravity dam design. Loadings that are not indicated should be included where applicable [2]:

- Load condition No. 1: unusual loading condition-construction
  - Dam structure completed
  - No headwater or tailwater
- Load condition No. 2: usual loading condition normal operating
  - Pool elevation at top of closed spillway gates where spillway is gated and at spillway crest where spillway is ungated
  - Minimum tailwater
  - Uplift
  - Ice and silt pressure, if applicable

The procedure of the design of a solid gravity dam involves the determination of theoretical profile initially and then the modification from practical point of view. The basic modifications required in the theoretical profile are:

- The sufficient freeboard is provided to avoid over flow from the dam. The requirement of free board is decided from the wave action created at the water surface. The minimum free board should be provided as 0.9 m.

The sufficient top width is provided which is required for the provision of road above the dam for inspection purposes. Due to above provisions, extra material is required at the top of the dam, which results in shifting of resultant towards the heel in reservoir empty condition and chances of development of tension at the toe. To avoid this tension, base width of the dam is increased at the upstream side and upstream batter is provided. Hence material is increased on the upstream side of the dam. In reservoir full condition, the resultant remains in middle third portion due to provision of top width and the section remains quite safe, hence the material from the downstream side may be removed to bring resultant in the outer middle third point. The material required in modification of theoretical profile consists of the material required at the top plus the material required at the upstream bottom minus material removed from downstream side. The net material required is a function of top width. Hence a particular top width is to be decided for which the net material required is the minimum. This top width is known as the optimal top width [3]. Creager [4] had proposed that the economical top width of gravity dam can be adopted as 14% of height of the dam. He had not considered earthquake forces. Several researchers have studied genetic algorithm in engineering application. In the study of Sarabian and Lee [5], Non-oriented case of Two-Dimensional Rectangular Bin Packing Problem (2DRBPP) was investigated. The objective of this problem was to pack a given set of small rectangles, which may be rotated by 90°, without overlaps into a minimum numbers of identical large rectangles. Aim was to improve the performance of the MultiCrossover Genetic Algorithm (MXGA) proposed from the literature for solving the problem. Rayner [6], proposed a genetic semi-supervised clustering technique as a means of aggregating data stored in
multiple tables to facilitate the task of solving a classification problem in relational database. This algorithm is suitable for classification of datasets with a high degree of one-to-many associations. It was shown in the experimental results that using the reciprocal of Davies-Bouldin Index for cluster dispersion and the reciprocal of Gini Index for cluster purity, as the fitness function in the Genetic Algorithm (GA), finds solutions with much greater accuracy.

The main objective of Al Rahedi and Atoum [7] study’s was to propose a new representation method of chromosomes using upper triangle binary matrices and a new crossover operator to be used as a heuristic method to find near-optimum solutions for the Traveling Salesman Problem (TSP). The outcomes obtained from running the proposed genetic algorithm on several TSP instances taken from the TSPLIB had showed that proposed methods found optimum solution of many TSP benchmark problems and near optimum of the others. Optimal operation of single and a cascade hydro-electricity reservoirs systems were analyzed by Asfaw and Saiedi [8] with using genetic algorithm and excel optimization solver and the results were comparatively analyzed. The objective function was to minimize the difference between actual and installed generation capacity of plants. The state transformation equation (the equation of water balance), the minimum and maximum stage and turbine releases were taken as constraints. The results showed that the release policy of genetic algorithm was better than that of excel optimization solver in two ways: greater electricity generation and convenience of the operation.

Other related topics can be traced on: Kumar and Vidivelli [9]; Tessa [10]; Eldrandaly [11]; Haut et al. [12]; Yedjour et al. [13]. This study deals with the determination of the optimal top width of gravity dam by a Genetic Algorithms in which the top width is assumed as percentage of the height of dam. For this top width, the extension of bottom width at upstream side is determined for no tension at the toe in reservoir empty condition. The top width is extended in upstream side such that tension should not develop at the toe in reservoir empty condition. The bottom width is reduced in the downstream side to save the construction material and precaution is taken that the tension should not develop at the heel in reservoir full condition. The bottom width is extended in the downstream side such that tension should not develop at the heel in reservoir full condition. The top width is assumed as percentage of the height of dam. In general, GAs approach the entire design space randomly and then improve the found design points by applying genetics based principles and probabilistic selection criteria [14]. Although a large number of modified algorithms are available, a GA typically proceeds in the following order:

- Start with a finite population of randomly chosen chromosomes (“design points”) in the design space. This population constitutes the first generation (“iteration”)
- Evaluate their fitness (“function value”)
- Rank the chromosomes by their fitness
- Apply genetic operators (mating): reproduction (reproduce chromosomes with a high fitness), cross-over (swap parts of two chromosomes, chosen based on their fitness to create their offspring) and mutation (apply a random perturbation to parts of a chromosome). All of these operators are assigned a probability of occurrence
- Assemble the new generation from these chromosomes and evaluate their fitness
- Apply genetic mating as before and iterate until convergence is achieved or the process is stopped

As can be seen above, the primary usefulness of the GA is that it starts by sampling the entire design space, possibly enabling it to pick points close to a global optimum. It then proceeds to apply changes to the ranked individual design points, which leads to an improvement of the population fitness from one generation to another. To ensure that it doesn’t converge on an inferior point, mutation is randomly applied, which perturbs design points and allows for the evaluation and incorporation of remote points.

The main advantages of GAs are:

- The nature of the optimization model does not need to be known. This makes GAs very interesting for complex problems or for users inexperienced in gradient-based optimization techniques
- The optimization model and its constraints do not have to be continuous or even real values. No simplification of a problem is necessary to accommodate it to a particular algorithm (e.g., linearization)
- They are readily available and easily implemented

The main disadvantages are:

A large number of parameters need to be set. This is simplified by information from literature, but problem-specific adjustments might need to be made.

Due to the comparatively very large number of function calls, GAs require significant computational resources. This makes them unattractive for optimization problems with computationally demanding analyses.

### II. MATERIALS AND METHODS

The elementary or theoretical profile is determined and then it is modified for the practical consideration. The bottom width is extended in upstream side such that tension should not develop at the toe in reservoir empty condition. The bottom width is reduced in the downstream side to save the construction material and precaution is taken that the tension should not develop at the heel in reservoir full condition. First the top width is assumed as percentage of the height of dam. For this top width, the extension of bottom width at upstream side is determined for no tension at the toe in reservoir empty condition.

Similarly the reduction in the bottom width is determined at the downstream side for no tension at the heel in reservoir full condition. The top width (a) is assumed as percentage of the water depth (h). By referring to Fig. 1, for the selected top
width, the bottom width of upstream batter (x) is assumed as a fraction of top width \( (a/x_1) \) and reduction in bottom width \( (y) \) in the downstream is also assumed as a fraction of top width \( (a/y_1) \). By changing values of \( x_1 \) and \( y_1 \), the values of width \( x \) and \( y \) are determined such that the dam section is safe for reservoir empty as well as for reservoir full conditions [3].

For various values of top width, the final profile of the dam section is obtained. The top width, corresponding to the minimum cross section of profile will be the optimal top width section.

In the present study, GA Optimization for Excel software is used for the design of dam corresponding to the optimal top width.

**Problem definition:** The optimization problem introduced above was implemented as a minimization of the cross section of a proposed gravity dam. All parameters, design variables, the objective function and all constraints have been inserted into an Excel spreadsheet for ready processing. The problem was defined as follows (Fig. 1):

The height of dam = \( H \) in meter
The height of water surface = \( h \) in meter.
The base width of the profile from no tension criteria is given by:

\[
b = \frac{h}{(\sqrt[3]{S} - 1)}
\]

Where:
\( S \) = Specific gravity of dam material
\( a \) = Top width provided (in meter)
\( f \) = Free board (in meter)
\( h_1 \) = The depth up to which vertical upstream face is provided (in meter)

This is given by:

\[
h_1 = H - h_2 - f
\]

\( h_2 \) = The height of upstream batter from the base (in meter)
\( x = a/x_1 \) = The width of upstream batter
\( h_3 \) = The height of downstream sloping face from the base

In triangle ECD:

\[
\frac{h}{h_3} = \frac{b_1}{(b_1 - a)} \Rightarrow a = b_1(1 - h_3)/h
\]

\( y = a/y_1 \) = The base width reduced in the downstream

III. RESULTS

*Calculation of moments:* With reference to Fig. 1, forces acting, lever arm from the toe and moments about toe are tabulated in Tables 1 and 2.

In the Table I;
\( S = \) specific gravity of dam material=2.4, \( w = \) specific weight of water in N/m\(^3\)=9810, \( M_1, M_2, \ldots \) = moments of corresponding forces about the toe, vertical \( (d) = \) vertical downward forces, vertical \( (u) = \) vertical upward forces.

**Computation of eccentricity:**

a) Reservoir empty condition:

\[
\bar{X} = \frac{M_1 + M_2 + M_3}{w_1 + w_2 + w_3}
\]

\[
e_d = \bar{X} - B/2 \quad , \quad e_d \leq B/6
\]

Where \( \bar{X} \) is the distance between the point of intersection of the resultant with the base and dam toe and \( e_d \) is the distance between the centroid of the area of the base and the point of intersection of the resultant with the base, or eccentricity of loading in reservoir empty condition.

b) Reservoir full condition:

\[
\bar{X} = \frac{M_1 + M_2 + M_3 + M_4 + M_5 - M_6 - M_7}{w_1 + w_2 + w_3 + w_4 + w_5 - U_1}
\]

\[
e_f = \frac{B}{2} - \bar{X} \quad , \quad e_f \leq B/6
\]

Where \( e_f \) is the distance between the centroid of the area of the base and the point of intersection of the resultant with the base, or eccentricity of loading in reservoir full condition.
Fig. 1 Basic loading conditions in concrete gravity dam design

TABLE I

<table>
<thead>
<tr>
<th>Forces</th>
<th>Expression</th>
<th>Nature</th>
<th>Lever arm</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of dam (w₁)</td>
<td>0.5h₁ x Sw</td>
<td>Vertical (d)</td>
<td>B – 2x / 3</td>
<td>M₁</td>
</tr>
<tr>
<td>Weight of dam (w₂)</td>
<td>aHSw</td>
<td>Vertical (d)</td>
<td>(b₂ – a) + 0.5a</td>
<td>M₂</td>
</tr>
<tr>
<td>Weight of dam (w₃)</td>
<td>0.5(b₃ – a)h₃Sw</td>
<td>Vertical (d)</td>
<td>2 / 3(b₃ – a)</td>
<td>M₃</td>
</tr>
<tr>
<td>Weight of water supported</td>
<td>h₁ x w</td>
<td>Vertical (d)</td>
<td>b₁ + 0.5x</td>
<td>M₄</td>
</tr>
<tr>
<td>by upstream face (w₄)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weight of water supported</td>
<td>0.5hₓ x w</td>
<td>Vertical (d)</td>
<td>bₓ + 2x / 3</td>
<td>M₅</td>
</tr>
<tr>
<td>by upstream face (w₅)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hydrostatic pressure (P₁)</td>
<td>0.5h²w</td>
<td>Horizontal</td>
<td>h / 3</td>
<td>M₆</td>
</tr>
<tr>
<td>Uplift pressure (U₁)</td>
<td>0.5h₁Bw</td>
<td>Vertical (u)</td>
<td>(2 / 3)B</td>
<td>M₇</td>
</tr>
</tbody>
</table>

TABLE II

<table>
<thead>
<tr>
<th>Forces</th>
<th>Value (N/m)</th>
<th>Nature</th>
<th>Lever arm (m)</th>
<th>Moment (N.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of dam (w₁)</td>
<td>52997.33</td>
<td>Vertical (d)</td>
<td>60.10</td>
<td>3185231.67</td>
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<tr>
<td>Weight of dam (w₂)</td>
<td>18299338.56</td>
<td>Vertical (d)</td>
<td>54.69</td>
<td>1000726837.26</td>
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<tr>
<td>Weight of dam (w₃)</td>
<td>39401996.11</td>
<td>Vertical (d)</td>
<td>33.06</td>
<td>1302538138.31</td>
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<tr>
<td>Weight of water supported</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>by upstream face (w₄)</td>
<td>223865.02</td>
<td>Vertical (d)</td>
<td>60.24</td>
<td>13484890.58</td>
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<tr>
<td>Weight of water supported</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>by upstream face (w₅)</td>
<td>24525.00</td>
<td>Vertical (d)</td>
<td>60.39</td>
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<td>Hydrostatic pressure (P₁)</td>
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<td>Horizontal</td>
<td>25.00</td>
<td>689765625.00</td>
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<td>Uplift pressure (U₁)</td>
<td>22330748.25</td>
<td>Vertical (u)</td>
<td>40.47</td>
<td>903680720.18</td>
</tr>
</tbody>
</table>

Excel file: Design values (gene values) are given to Excel and function values (fitness values) and constraint values are read from Excel interface. As can be seen in Fig. 2, an Excel file, which contains the calculation model, can be selected and cell references for the function value, all design variables and all constraints can be specified. On another tab, the user can modify the given GA parameters and then on a third tab, the user can run the GA algorithm and capture its output. All
design variables can have lower and upper limits. If during the GA process, a gene reaches or overshoots any of these limits its value is set to the limit itself. The following conclusions were drawn from the parametric study:

- Increasing the number of chromosomes per population directly increased the accuracy of the solution. This was most pronounced when the number changed from 8-50. Changing it to 100 did not yield a significant improvement. Evidently, a higher number of chromosomes provide a higher chance of starting closer to the optimum. Since the number of chromosomes generally determines the number of function calls, it should be kept to a minimum.

- Increasing the cross-over probability from 0.6-0.9 also yielded an improved performance. A cross-over probability of 1, however, decreased the accuracy significantly.

- Probability of mutation was found to be well performing at the 0.1 level. Although this is quite high compared to the recommended values for the mutation probability (0.01-0.02), the nature of the test function and the small target (peak) area explain the necessity of having this parameter at high levels. When this parameter was chosen at too high level (0.5 for example), it was observed that it led to an unstable convergence and necessitated a significantly higher number of generations before convergence.

- The effect of maximum generations and selecting the best performing individuals for the starting generation of the actual runs was found to be significant. Comparing only the number of preliminary runs, a number of 2 showed the lowest and 8 the best performance. When the number of generations per preliminary run was set to a higher value (50), performance was increased. Similar to the total number of generations, however, the preliminary run generations also contribute to the total number of function calls and should be kept to a minimum.

Application of GA for optimal top width of gravity dam: The optimization of the of gravity dam dimensions was performed using the values given in Table 3. Target cell in Excel spreadsheet was selected as dam dead weight (W) and decision variable were selected as a, x and y that after optimization, their values obtained 10.2, 0.9 and 3.6 m respectively.

<table>
<thead>
<tr>
<th>Optimization of the of Gravity Dam Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>h (m)</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>75</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>h1 (m)</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>62.93</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>59.78</td>
</tr>
</tbody>
</table>

IV. DISCUSSION

This study deals with the determination of the optimal top width of gravity dam by a Genetic Algorithms in which the top width is taken as a function of water depth. First the top width is assumed as percentage of the height of dam. For this top width, the extension of bottom width at upstream side is determined for no tension at the toe in reservoir empty condition. The net required material is calculated for the dam section in which no tension is developed anywhere in the dam section. The proposed methodology can be used very conveniently to determine the safe and economical dam section corresponding to the optimal top width. The optimization problem introduced above was implemented as a minimization of the cross section of a proposed gravity dam. All parameters, design variables, the objective function and all constraints have been inserted into an Excel spreadsheet for ready processing. In Table 4, other optimization run for gravity dam cross section is presented. Based on table 1, ratio of is dam base (B) to dam height (H) is almost equal to 0.85, and ratio of dam top width (a) to dam height (H) is almost equal to 0.13.

<table>
<thead>
<tr>
<th>Optimization of the of Gravity Dam Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>H (m)</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>76.2</td>
</tr>
<tr>
<td>80</td>
</tr>
<tr>
<td>90</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

V. CONCLUSION

The design of a gravity dam is performed through an interactive process involving a preliminary layout of the structure followed by a stability and stress analysis. If the structure fails to meet criteria then the layout is modified and reanalyzed. This process is repeated until an acceptable cross section is attained. Modification could be suitable free board and suitable top width, that due to such provisions the weight of the dam is to be increased at the top portion, which creates instability of the section in reservoir empty condition and tension may be develop at the toe. Hence it becomes necessary to increase weight in the upstream and a batter is provided on the upstream side. In this study the design of gravity dam corresponding to the optimal top width can be carried out for any required height of the dam. The obtained design is the...
most economical and the safest in which no tension is developed anywhere in the dam section. Computation of optimal top width of gravity dam performed with genetic algorithm method by using Excel spreadsheet software.

REFERENCES