Probablistic Models

Keywords: Model, Probabilistic Model

Cholette & Cloutier

They proposed a model for real CSTR. It consists of an active perfectly mixed region and a completely dead region with no transfer of material to the other region and a certain fraction of the feed by passing both regions.

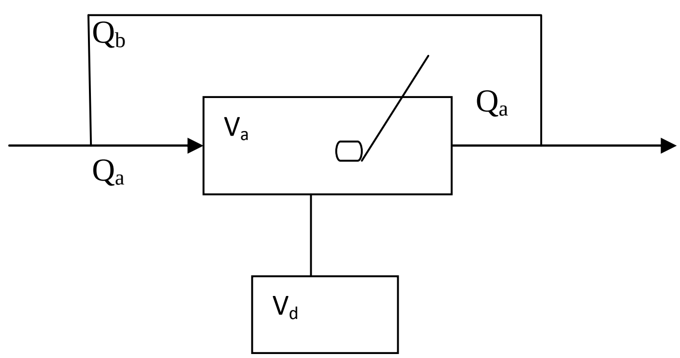

Fig. 16.1: Model of Real CSTR

Probabilistic Approach:

Consider that at t=0 a step input of the tracer is given to the reactor. Instead of determining the concentration of tracer in outlet stream assume that a tracer fluid element enters the system at t=0 either it may go in the by pass stream or to active region. It resides there. For a random time and leaves the system here there are three states or tracer fluid element where it can reside

State -1 Active region

State- 2 Outlet stream

State 3 By pass stream or region

On entering the vessel tracer fluid element may go to state 2 through state 1 or state 3 depending upon the initial probability distribution (EQ 16.1) or we can say that at any time tracer fluid element may be in any of the three states while timer passes continuously. So, logical description of the passage of tracer fluid element through the system may be considered as a discrete state continuous time stochastic process.

Assumptions:

1. Flow rate is steady so λ_i is constant (i=1. 3) where λ is defined as mean frequency of passage of element through the ith stage.
2. At time t element is in the ith stage then the probability that at time $(t+\Delta t)$ it leaces the ith stage and goes to newt i.e. stage 2 is $\lambda_1 \Delta t$
3. At time t tracer fluid element in the ith stage then the probability that at time $(t+\Delta t)$ it remains there is $(1- \lambda_1 \Delta t)$
4. Both the stages (1&3) are considered to behave like a CSTR
5. System considered is a closed vessel

Initial conditions:

$$P1(0) = \left.\begin{cases} a \; ; \; i=1 \\ 1-a \; ; \; i=3 \\ 0 \; ; \; \text{otherwise} \end{cases}\right\} \quad \ldots 16.1$$

a = fraction of feed going to active zone

$$= Q_a/Q$$

Our object is to compute P2(t) i.e. probability that the element has left the system P2(t) is the fact F9T) i.e. cumulative residence time of the system.

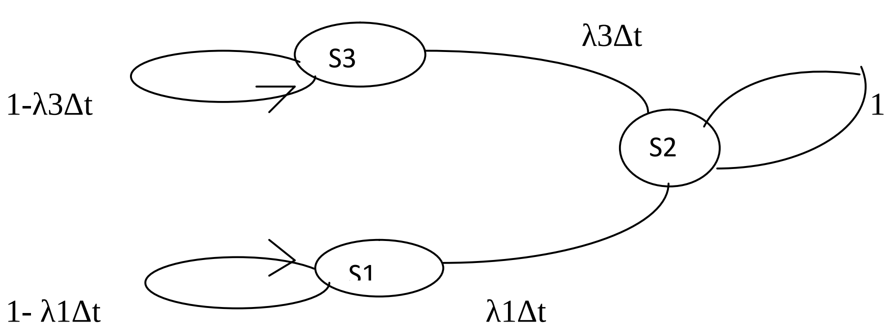

Fig. 16.2: Probabilities of Tracer Element

It is desired to evaluate P1(t+Δt) only possibility is that at time t tracer fluid element was in state 1 and in the transition it remain there. Hence

$$P1\ (t+\Delta t) = P1(t)\ (1-\lambda_1\Delta t)$$

Similarly

$$P3(t+\Delta t) = P3(t)\ (1-\lambda_3\Delta t)$$

$$P2(t+\Delta t) = P2(t) + \lambda_1 P1(t)\,\Delta t + \lambda_1 P3(t)\,\Delta t$$

$$\frac{dP_1(t)}{dx} = -\lambda_1 P_1(t)$$

$$\frac{dP_3(t)}{dx} = -\lambda_3 P_3(t)$$

$$\frac{dP_2(t)}{dx} = \lambda_1 P_1(t) + \lambda_3 P_3(t)$$

Take Laplace transformation

$$S\ L1(s) - a = -\lambda_1 L1(s)$$

$$sL3(s) - (1-a) = -\lambda_3 L3(s)$$

$$s\ L2(s) = \lambda_1 L1(s) + \lambda_3 L3\ (s)$$

$$L_2(s) = \frac{(1-a)\lambda_3}{s(s+\lambda_3)} + \frac{a\lambda_1}{s(s+\lambda_1)}$$

Taking inverse L.T.

$$P_2(t) = F(t) = (1-a)\left[1-a^{-\lambda_3 t}\right] + a\left[1-a^{-\lambda_1 t}\right]$$

It is reported in the literature that if some the fluid passes through the vessel in a time one tenth of mean residence time of the overall fluid for all practical purposes two fluid can be said to bypass the vessel. So for example if 0.1 is the fraction of the total fluid by passing the vessel then volume of by pass region should be less than or equal to 1/100 of the total volume of the vessel which is negligibly small Due to this reason volume of bypass stream has not been shown in fig 1 since λ_3 is the ratio of Q_b to volume of bypass region therefore here λ_3 may be assumed as approaching infinity. In case the volume of bypass region is of appreciable magnitude then it may be taken into account accordingly

$$Lim_{\lambda_3 \to \infty}\left[1 - e^{-\lambda_3 t}\right] = U(t)$$

Now Eq. 6 becomes

$$F(t) = (1-a)U(t) + a[\,1 - a^{-\lambda_1 t}]$$

$$F\,(\theta) = \frac{Q_b}{Q}\,U(\theta) + \frac{Q_a}{Q}\left[1 - e^{-\frac{Q_a}{Q}\frac{V}{V_a}\theta}\right]$$

Hence

$$E\,(\,\theta) = \frac{dF(\theta)}{d\theta} = \frac{Q_b}{Q}\,\partial(\theta) + \left(\frac{Q_a}{Q}\right)^2 \frac{V}{V_a}\left[e^{-\frac{Q_a}{Q}\frac{V}{V_a}\theta}\right]$$

Equations are identical to those obtained by deterministic theory.

Source:
http://nptel.ac.in/courses/103107096/16