

# Phase noise of diode laser in self-mixing interference

Tao Geng, Gang Li, Yuchi Zhang, Junmin Wang, and Tiancai Zhang

State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan, 030006, Shanxi, China  
[tczhang@sxu.edu.cn](mailto:tczhang@sxu.edu.cn)

**Abstract:** We investigate the phase noise of a diode laser based on the interferometric self-mixing effect. A detuned Fabry-Perot cavity converts the phase noise into intensity noise, and the noise is measured by the novel method as a function of the amount of feedback and the distance between the target and the laser front facet. Experimental results can be well explained by theory.

© 2005 Optical Society of America

OCIS codes: (140.2020) Diode lasers; (120.0120) Instrumentation, measurement and metrology.

---

## References and links

1. J. Hast, "Self-mixing interferometry and its applications in noninvasive pulse detection," doctoral dissertation to be presented with the assent of the Faculty of Technology, University of Oulu, for public discussion in Raahensali (Linnanmaa, 2003), p. 20.
2. P. G. R. King and G. J. Steward, "Metrology with an optical maser," *New Scientist* **17**, 180 (1963).
3. G. Giuliani, M. Norgia, S. Donati, and T. Bosch, "Laser diode self-mixing technique for sensing applications," *J. Opt. A: Pure Appl. Opt.* **4**, S283 (2002).
4. W. M. Wang, W. J. O. Boyle, K. T. V. Grattan, and A. W. Palmer, "Self-mixing interference in a diode laser: experimental observations and theoretical analysis," *Appl. Opt.* **2**, 1551 (1993).
5. L. Scalise, Y. Yu, G. Giuliani, G. Plantier, and T. Bosch, "Self-mixing laser diode velocimetry: application to vibration and velocity measurement," *IEEE Trans. Instrum. Meas.* **53**, 223 (2004).
6. F. Gouaux, N. Servagent, and T. Bosch, "Absolute distance measurement with an optical feedback interferometer," *Appl. Opt.* **37**, 6684 (1998).
7. S. Donati, G. Giuliani, and S. Merlo, "Laser diode feedback interferometer for measurement of displacements without ambiguity," *IEEE J. Quantum Electron.* **31**, 113 (1995).
8. N. Servagent, F. Gouaux, and T. Bosch, "Measurement of displacement using the self-mixing interference in a laser diode," *J. Opt.* **29**, 168 (1998).
9. P. J. de Groot and G. M. Gallatin, "Ranging and velocimetry signal generation in a backscatter-modulated laser diode," *Appl. Opt.* **27**, 4475 (1988).
10. G. Giuliani, S. B. Pietra, and S. Donati, "Self-mixing laser diode vibrometer," *Meas. Sci. Technol.* **14**, 24 (2003).
11. G. Giuliani and M. Norgia, "Laser diode linewidth measurement by means of self-mixing interferometry," *IEEE Photon. Technol. Lett.* **12**, 1028 (2000).
12. J. A. Armstrong, "Theory of interferometric analysis of laser phase noise," *J. Opt. Soc. Am.* **56**, 1024 (1966).
13. G. A. Acket, D. Lenstra, A. J. Den Boef, and B. H. Verbeek, "The influence of feedback intensity on longitudinal mode properties and optical noise in index-guided mode semiconductor lasers," *IEEE J. Quantum Electron.* **QE-20**, 1163 (1984).
14. H. W. M. Salemink and J. W. M. Biesterbos, "Optical stability of narrow stripe, proton-isolated AlGaAs double heterostructure lasers with gain guiding," *Appl. Phys. Lett.* **43**, 434 (1983).
15. Y. Yu, G. Giuliani, and S. Donati, "Measurement of the linewidth enhancement factor of semiconductor lasers based on the optical feedback self-mixing effect," *IEEE Photon. Technol. Lett.* **16**, 990 (2004).
16. S. Donati and G. Giuliani, "Analysis of the signal amplitude regimes in injection-detection using laser diodes," in *Physics and Simulation of Optoelectronic Devices VIII*, R. H. Binder; P. Blood, and M. Osinski, eds., *Proc. SPIE* **3944**, 639 (2000).
17. K. Petermann, *Laser Diode Modulation and Noise* (Kluwer Academic, Dordrecht, The Netherlands, 1988).
18. T.-C. Zhang, J.-P. Poizat, P. Grelu, J.-F. Roch, P. Grangier, F. Marin, A. Bramati, V. Jost, M. D. Levenson, and E. Giacobino, "Quantum noise of free-running and externally-stabilized laser diodes," *Quant. Semic. Opt.* **7**, 601 (1995).

## 1. Introduction

Laser interferometry has been used widely in industry and research. In recent years, a technique called self-mixing interferometry (SMI) [1] was developed, in which part of the output emitted beam is retroreflected or backscattered into the laser cavity and self-mixed with the original light. Under certain conditions, the frequency-shifted external light is mixed coherently with the original light; both the amplitude and the frequency of the diode laser are modulated, and one can see self-mixing interference by monitoring the photodiode included in the laser diode (LD) package.

In the 1960s it was found that the intensity modulation in the output of the laser was induced by external feedback [2]. The properties of the laser strongly depend on the parameters of optical feedback. The problems, such as mode-hopping, frequency instability, and noise-enhancing, would arise in the usual experimental environment. However, it is found that the self-mixing effect has been applied for optical measurement [3] and information processing [4]. Compared with traditional interferometry, the self-mixing configuration is simply related to the diode laser, the focusing optics, and the target under test conditions [5]. This method can be applied to almost all LDs. Due to its desirable features, such as high sensitivity, high accuracy, and contactless operation, SMI has led to various applications in the measurement of distance [6], displacement [7,8], velocity [9], and ranging [10,11]. For a well-stabilized diode laser, the phase noise, which changes with time in a random fashion [12], plays a very important role because the intensity noise has been found to be extremely small. Actually, the intensity fluctuation is very close to the shot noise limit when the diode laser operates far above the threshold. In this case, the phase noise becomes dominant and it directly determines the linewidth of the LD eventually [11]. Therefore, the phase noise of the LD is important in order to understand the noise characteristics of the LD. In this article, the phase noise in the SMI system is measured for the first time, to our knowledge, by using a detuned scanning Fabry-Perot cavity and a spectrum analyzer. Phase noise as a function of the amount of external feedback and the distance between the remote target and the laser front facet are measured, and the results agree with the theory. The proposed approach provides a novel method for determining the phase noise in self-mixing interference. Due to its simplicity, reliability, and compactness, this method can be used in other systems to measure phase noise.

## 2. Basic idea

Figure 1 shows the schematic configuration of self-mixing interference. The characteristic of the laser output depends on both the target-distance and the amount of optical feedback. The system can be modeled as a three-mirror Fabry-Perot cavity.  $M_1, M_2$  are the laser cavity mirrors, and their amplitude reflectivities are  $R_1, R_2$ , respectively. Here, for simplicity, assume that  $R_1 = R_2 = R$ ,  $S$  is the distance between the laser front facet and the remote target  $M_{ext}$ , also called the length of external cavity.

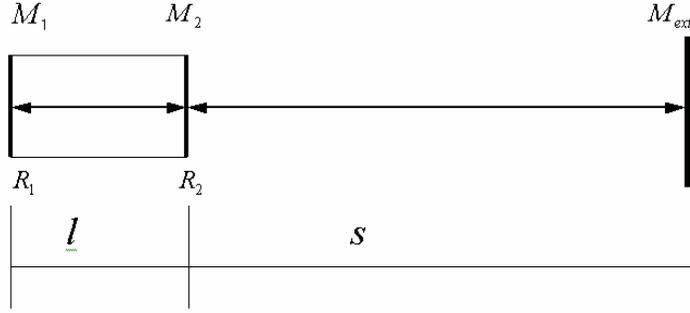


Fig. 1. Schematic figure of self-mixing interference.

As discussed by Gerard, the properties of the LD depend on the delay of the retroreflected light and the feedback parameter [13]  $C$

$$C = f \frac{S}{l} (1-R) \left[ (1+b^2) \frac{r}{R} \right]^{1/2}, \quad (1)$$

where  $l$  is the effective diode cavity length;  $R$  is the power reflectivity of the laser mirrors;  $r$  is the ratio of the power reflected back into the laser to the power emitted by LD;  $b$  is the ratio of real to imaginary parts of the variation in the complex refractive index due to the injected carriers, which is close to  $-4$  for GaAlAs materials [14]; and  $f$  is the correct factor. The phase noise of the LD in self-mixing interferometry is given by [13]

$$\begin{aligned} \langle (\Delta P)^2 \rangle &= \frac{16\omega_0^2 P_0^2}{c^2 \Gamma_0^2} \left( \frac{J/J_{th}}{J/J_{th} - 1} \right)^2 \cdot \frac{\gamma^2 \sin^2 [(\omega_0 + \Delta\omega)]}{\{1 + C \cos [\varphi_0 - (\omega_0 + \Delta\omega)\tau]\}^2} \\ &\cdot \langle (\Delta L)^2 \rangle, \end{aligned} \quad (2)$$

where  $\gamma = f \frac{c}{2l} (1-R)(r/R)^{1/2}$ ,  $\omega_0$  is the laser angular frequency,  $\varphi_0$  is the initial phase,  $\Delta\omega$  is the feedback-induced shift multiplied by  $2\pi$  with respect to  $\omega_0$ ,  $\Gamma_0$  is the diode cavity intensity loss,  $c$  is the velocity of light,  $\tau$  is the diode cavity intensity loss,  $J$  is the current density in the active layer with  $J_{th}$  as its value at the threshold of laser operation, and  $\sqrt{\langle (\Delta L)^2 \rangle}$  represents the fluctuations of the external cavity length. It was discussed previously that the region for  $C > 1$  is substantial, since in this region the interferometric signal exhibits nonsinusoidal (i.e., high-order harmonics appear) and fast switchings every  $\lambda/2$  target displacement [15]. This behavior is very useful in easily detecting the displacement without direction ambiguity [16,17]. Thus, the phase noise properties of the LD in this region become particularly significant for the self-mixing effect, and we will focus our investigation in this region. The phase noise decreased with the increasing of  $r$ , and when the distance  $S$  is large enough, the phase noise is proportional to the distance  $S$  in this status [11].

The measurement of phase noise is not trivial in this process. We used a phase-amplitude converter, i.e., a detuned empty Fabry-Perot cavity [18] in the experiment, as shown in Fig. 2.  $E_{in}, E_{out}$  are the amplitudes of the input and output field which is back-reflected from the cavity, respectively. The empty cavity is formed by two curved mirrors with amplitude reflection coefficients,  $r_1, r_2$ , respectively. Assume that  $p, q$  are the amplitude and phase quadrature components of field in Fourier space. The noise power, which is normalized to the shot noise at an analysis frequency  $\Omega$  of the back-reflected field from the cavity, can be expressed as follows [19]

$$S(\Omega) = \left| \frac{1}{2} \left( \frac{1+r_1^2 r_2^2 - 2r_1 r_2 \cos \phi}{r_1^2 + r_2^2 - 2r_1 r_2 \cos \phi} \right)^{1/2} \cdot \left\{ \frac{r_2 \exp(i\phi) - r_1}{1-r_1 r_2 \exp(i\phi)} \frac{r_2 \exp[-i(\phi-\Omega)] - r_1}{1-r_1 r_2 \exp[-i(\phi-\Omega)]} + \frac{r_2 \exp(-i\phi) - r_1}{1-r_1 r_2 \exp(-i\phi)} \frac{r_2 \exp[i(\phi+\Omega)] - r_1}{1-r_1 r_2 \exp[i(\phi+\Omega)]} \right\} \right|^2 + \left| \frac{1}{2} \left( \frac{1+r_1^2 r_2^2 - 2r_1 r_2 \cos \phi}{r_1^2 + r_2^2 - 2r_1 r_2 \cos \phi} \right)^{1/2} \cdot \left\{ \frac{r_2 \exp(i\phi) - r_1}{1-r_1 r_2 \exp(i\phi)} \frac{r_2 \exp[-i(\phi-\Omega)] - r_1}{1-r_1 r_2 \exp[-i(\phi-\Omega)]} - \frac{r_2 \exp(-i\phi) - r_1}{1-r_1 r_2 \exp(-i\phi)} \frac{r_2 \exp[i(\phi+\Omega)] - r_1}{1-r_1 r_2 \exp[i(\phi+\Omega)]} \right\} \right|^2 |q|^2, \quad (3)$$

where  $\phi$  is the round-trip delay of phase in the cavity. The frequency has been normalized to the bandwidth of the cavity. The noise power includes amplitude and phase noise, but when the rear mirror is highly reflected, the phase noise dominates over the total noise of the light back-reflected from the cavity. Numerical simulation indicates that for our cavity mirror with  $r_1 = 95.5\%$ ,  $r_2 = 98.7\%$ , the phase noise contributes 99.93% of the overall noise power whereas intensity noise, which is close to the shot noise limit, is much lower than the phase noise, thus the measured noise could be considered as the phase noise of the input field.

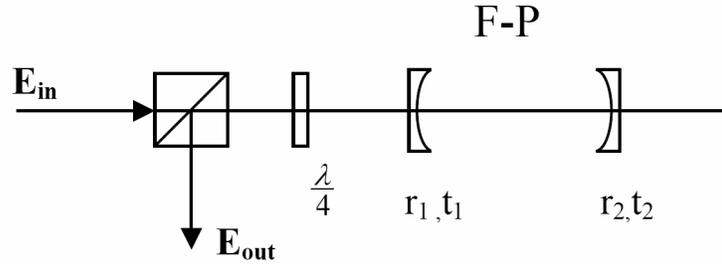


Fig. 2. Detuned Fabry-Perot cavity for phase noise measurement.

### 3. Experimental setup and results

The experimental setup is shown in Fig. 3. The laser diode we have used is a multi-quantum-well GaAlAs laser diode (Model HL-7851G). The center wavelength is 780 nm in the case of free-running. The threshold is 45mA, and the rear facet reflection coefficient is about 30%.

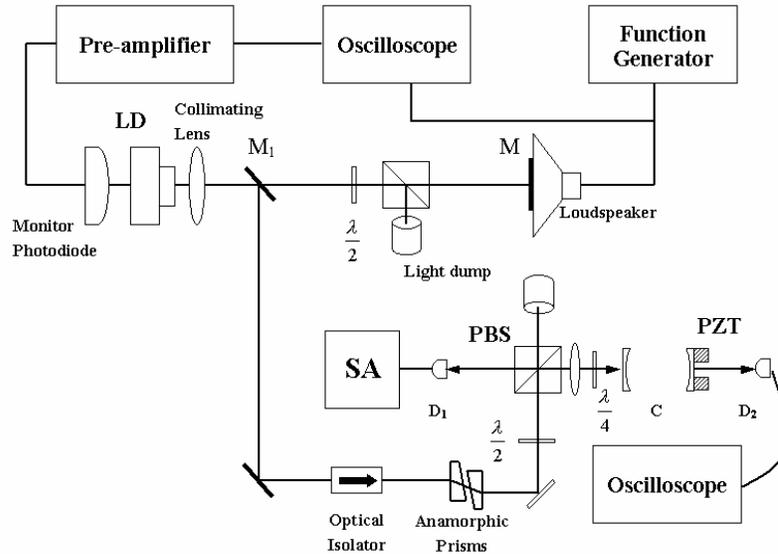


Fig. 3. Experiment setup for self-mixing interference. LD: laser diode from Hitachi (model HL-7851G); M: feedback mirror; SA: spectrum analyzer; PZT: piezoelectric transducer; C: Fabry-Perot cavity; D: photodetector.

The LD is temperature stabilized, and the output beam is collimated by a collimating objective lens with a numerical aperture of 0.5. The beam is split by a beam splitter  $M_1$  with a transmission coefficient of about 18%. The transmission light goes to a mirror  $M$  with high reflectivity, which is stuck onto a loudspeaker. When the loudspeaker is modulated with a sinusoidal wave signal, the output beam will be retroreflected into the LD cavity, and the self-mixing interference will occur. In order to make the laser source work in an optimum attenuation range that ensures operation with  $C > 1$ , a half-wave plate combined with a polarized beam splitter (PBS) is used for controlling the amount of feedback. The actual  $C$  in the experiment is about 4.

The reflection beam from  $M_1$  is used for phase noise measurement. It goes through an optical isolator (IsoWave, Inc.) with an isolation of 30dB and a transmission coefficient of 90%, anamorphic prisms, and then reaches a Fabry-Perot cavity with a length of about 104mm. The leakage from the rear mirror of the cavity allows us to monitor the intracavity intensity, which is useful for optical alignment and mode matching. This rear mirror is mounted on a piezoelectric transducer (PZT) so that the length of the cavity can be scanned.

The beam back-reflected from the cavity is measured by a photodetector (PD) (Hamamatsu S5972). The RF signal from the detector is sent to a spectrum analyzer (HP5890D), and the linear response of the detector has been checked. It is shown that the light power impinging on the PD could reach up to  $3mW$  before it is saturated. So, an attenuator is actually used in front of the detector.

In the first round of the experiment, we occasionally set the operating current of the LD to 74.5mA and the output power is about 19.5mW. The external cavity length is about 0.5m and the length modulation is about  $2\mu m$  by driving the loudspeaker with a sinusoidal wave signal.

Figure 4 shows the noise spectra when there is no external feedback (just by blocking the back-reflected beam from the loudspeaker). Trace A shows an M shape of the noise when the cavity is scanned across the resonance, which can be understood from the quadrature rotation from a detuned cavity [18]. The width of the M shape profile depends on the scanning time of the cavity as well as the spectrum analyzer, but the maximum of the peak, which corresponds to

the phase noise at certain power of light impinging on the PD, *does not change*. Trace B is the fitting result based on Eq. (3). The discrepancy between the fitting and the experiment trace at the resonance is obvious. This discrepancy was actually observed in Ref. [18]. There are some reasons for this discrepancy. First, the peak at cavity resonance is very narrow. Second, the laser line width and the general stability of the setup cause the jittering of the sharp peak and there is a finite resolution of spectrum analyzer that smoothes out this peak. All these prevent the measured noise at cavity resonance to go down to zero dB. Trace C and D are the shot noise and electronic noise, respectively. The typical analysis frequency is 50MHz, which is chosen according to the response of the detector. Noise at other analysis frequencies is also measured, and we get similar results. The result shows that for the diode laser under free-running, the phase noise is more than 100 times higher than the intensity noise.

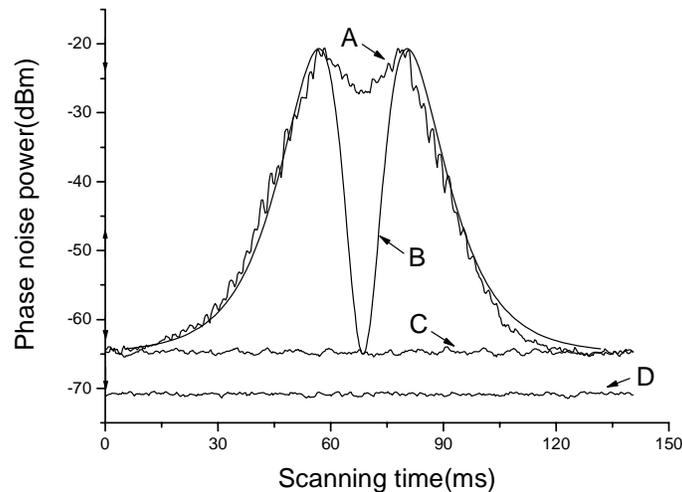


Fig. 4. The noise spectrum of LD without external feedback. Analysis frequency:  $50\text{MHz}$ . A: noise when the cavity is scanned across the resonance; B: the fitting based on the Eq.(3); C: shot-noise level when cavity is far from resonance; D: the electronic noise of the PD. The parameters of the SA:  $\text{RBW}=300\text{kHz}$ ;  $\text{VBW}=300\text{Hz}$ ; Scanning time:  $120\text{ms}$ .

Figure 5 shows the result when the self-mixing interference occurs. Due to the effect of SMI, the feedback phase varies because of the modulation and the oscillations of the phase noise power spectra can be seen. Typical traces are shown as trace A and B. The exact profile of the trace depends on the scanning of the cavity, the scanning of the loudspeaker, and the spectrum analyzer's scanning time; however, the maximum and minimum of the peaks corresponding to the phase and intensity of the measured light, respectively, *are stable* (see trace A and B) similar to what is shown in Fig. 4. It is clearly shown that the phase noise is reduced about 20dB compared with the case in free running, and this noise reduction depends on the feedback amount and the length of the external cavity.

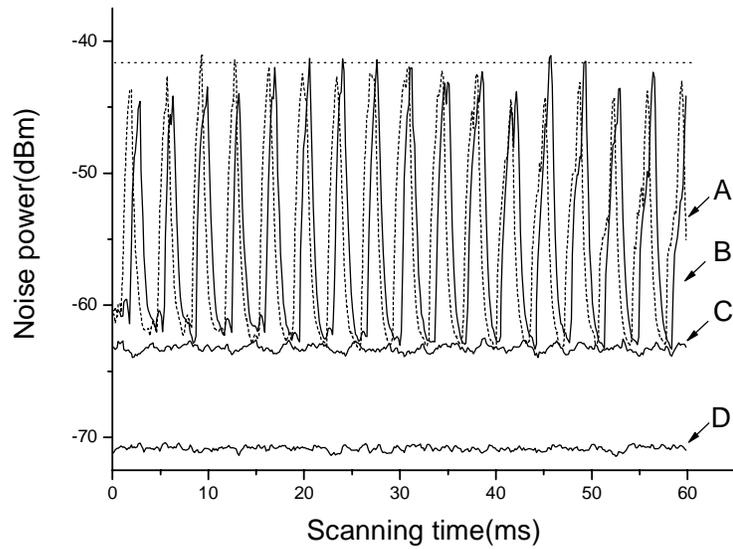


Fig. 5. Noise spectra with SMI at 50MHz. A and B are typical traces when the scanning time of SA is  $120ms$  ; C: shot-noise level; D: electronic noise level. The parameters of the SA:  $RBW=300kHz$  ;  $VBW=300Hz$  .

Thus, we can get the phase noise in the average range by frequent measurements under certain conditions. Figure 6 shows the result of the phase noise when the amount of the feedback varies. The dots are the experimental results. Each dot comes from 50 measurements as was done in Fig. 5. The solid curve is the fitting result. It verifies that the phase noise decreases with the increasing of the amount of feedback when the distance between the target and the LD is fixed. The analysis frequency is still  $50MHz$  .

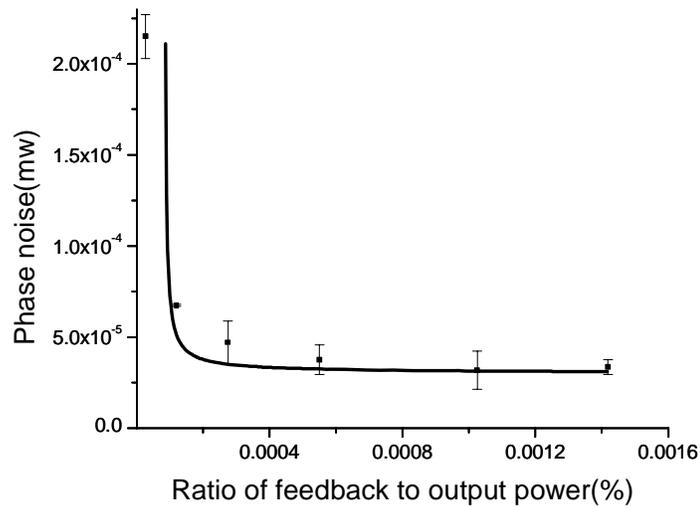


Fig. 6. Phase noise versus the amount of feedback. External cavity length: 0.5m.

We also measure the phase noise as a function of external cavity length while keeping the amount of feedback unchanged. The result is shown in Fig. 7. The driving current is  $120mA$  here. It is shown that phase noise increases when we increase the distance [11]. However, the variation of the phase noise becomes larger when the distance is greater, since the mechanical instability of the system and the variation of the feedback ratio is uncontrollable in the large distance.

Note that Eq. (2) contains many parameters and some of them are not well known. But the complex relationship between the measured noise power and the feedback ratio  $r$  can be simplified according to Eq. (2) as

$$\langle(\Delta P)\rangle_m = A \left( \frac{r}{(1-B\sqrt{r})^2} \right)^{1/2}. \quad (4)$$

All the other parameters in Eq. (2) are included in  $A$  and  $B$ . The actual measured noise power is also dependent on the various losses, the quantum efficiency of the detector, and the amplification of the electronics. In Fig. 5, the fitting parameter is selected as  $A=0.004$  and  $B=200$ . The relation between noise power and the external cavity length in Fig. 7 was extensively discussed by Giuliani, and the linear dependence is expected [11]. These results can be understood properly.

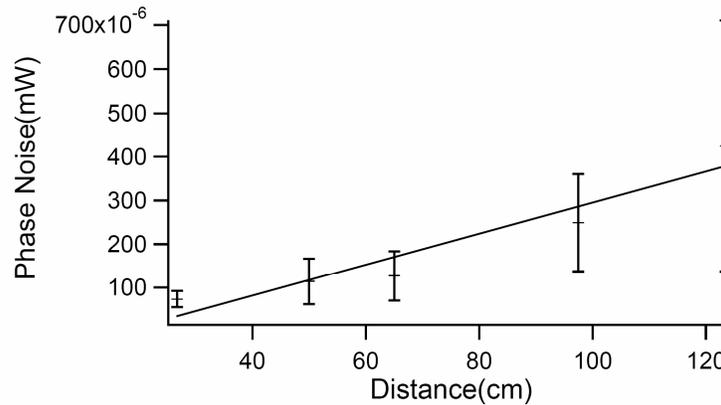


Fig. 7. Phase noise via external cavity length. The feedback amount is 0.006%. Driving current is  $120mA$ .

#### 4. Conclusions

We demonstrated a novel method of determining the phase noise of a LD in self-mixing interference by using a scanning detuned Fabry-Perot cavity. The key point is to figure out the phase noise from the disordered noise spectra for a certain amount of light from the SMI system. The phase noise is strongly affected by the external cavity length and the amount of feedback. We show that the phase noise increases as the external cavity length increases, while the phase noise reduces with the increasing of the amount of feedback. The investigation of the phase noise can help us to understand the noise properties of the SMI process. The configuration of investigating the phase noise reported here can be extended to studying the amplitude/phase noise relation, linewidth of the diode laser, and the chaotic behaviors on different situations.

## **Acknowledgments**

The authors thank Dong Yabing and Li Liping for their former work and gratefully appreciate Xu Zhiyong and J.P.Poizat for enlightening discussions. This research was supported by National Natural Science Foundation of China under grants 10434080 and 10374062, funding from the MOE, Research Funds for Returned Scholar Abroad, and Youth from Shanxi Province (No. 20031002).