Nuclear Physics

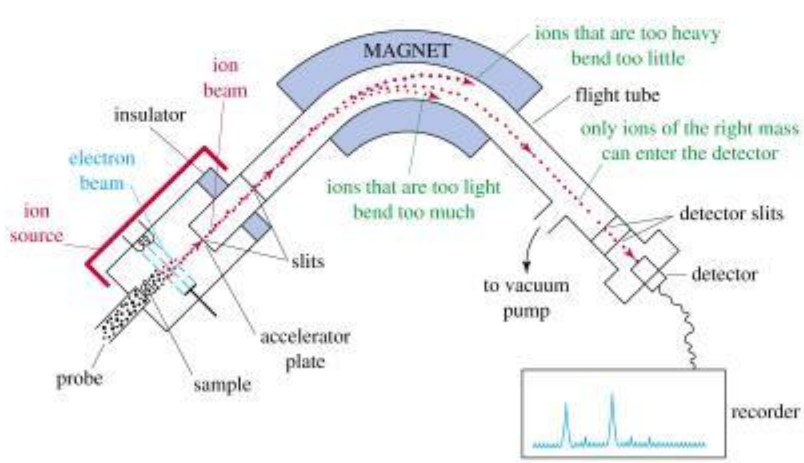
Explain how the radii of nuclei can be determined by charged particle scattering experiments.

Use of energy conservation for determining closest-approach distances for Coulomb scattering experiments is sufficient.

Essentially we toss charged particles towards a nucleus and measure what angles they come off and at and then calculate backwards how close they must have been to the nucleus… Since the nucleus is positively charged it will exert a force on the charged particle and thus alter its trajectory. Often the mass of the nucleus is much larger than that of the charged particle so when the charged particle "collides" the change in momentum of the nucleus can be ignored. Thus the particle goes in with a known amount of kinetic energy and will come out with the same amount of kinetic energy. As the charged particle gets close to the nucleus the electric potential energy of the charged particle will change. The math is not particularly simple, but using some fancy algebra and conservation of energy we can calculate the "distance of closest approach." If we keep giving the particle more energy we can get closer and closer thus getting better approximations of the size of the nucleus. Check out hyperphysics for a more complete discussion and a derivation.

Describe how the masses of nuclei can be determined using a mass spectrometer.

Students should be able to draw a schematic diagram of the mass spectrometer but the experimental details are not required. Students should appreciate that nuclear mass values provided evidence for the existence of isotopes.



Isotopes of a given element differ only by the number of neutrons in the nucleus. Thus different nuclides have different masses. Therefore we should be able to sort the isotopes by their mass…

This is done using what is called a mass spectrometer. First the atoms are ionized so that they have a net electric charge. This allows us to do a couple of things.

Picture Stolen from:http://en.wikipedia.org/wiki/Mass_spectrometry

First it allows us to accelerate the ions using an electric potential. The ions then pass through a uniform magnetic field, that is perpendicular to the path of the ions. *In the picture above the magnetic field would be perpendicular to the page.* Since the ions are charged and moving relative to the magnetic field they feel a tug. The tug moves then in a circular path. The more massive the ion the less it accelerates and the large its circular path. The less massive an ion is the more it accelerates and the smaller its circular path. Thus the isotopes have been separated by mass.

The force on the ions is:

(1)
$$F=qvB$$

Where q is the charge of the ion, v is the velocity of the ion and B is the strength of the magnetic field. Then using Newton's 2nd law:

(2)
$$F=ma$$

Combining the two:

(3)
$$ma=qvB$$

We find that the mass is:

(4)
$$m=qvB / a$$

Where the acceleration is centripetal and given by the following equation:

(5)
$a = v^2 / r$

Where v is the velocity of the particle and r is the radius of the circular path. Thus the mass is:

(6)
$m = qBr/v$

Its worth noting that the mass spectrometer actually measures the mass to charge ratio, not just the mass, however if you are dealing with atoms of the same element they will all have the same charge when ionized and thus you are effectively measuring the mass.

Describe one piece of evidence for the existence of nuclear energy levels.

When radioactive nuclei decay by gamma decay the energies of the gamma rays is distinctive of the nuclei. As the nucleus de-excites is does so very much in the same way that an electron around an atom de-excites. The nucleus has orbitals and energy levels, so when it drops from one energy level to a lower energy level energy is released in the form of a gamma ray (high energy photon). The energy of the gamma ray is equal to the energy different between the two nuclear energy levels.

(7)
$E_\gamma = E_2 - E_1$

Describe both $\beta+$ and $\beta-$ decay, including the existence of the neutrino and the antineutrino.

Students should know that β energy spectra are continuous and that the neutrino was postulated to account for the missing energy and momentum.

Beta Decay – There are two types of beta decay, positive and negative. Positive beta decay is the process by which a neutron decay into a proton and an electron. The neutron is made of three quarks, one up quark and two down quarks. One of the down quarks is converted to an up quark by the emission of a W boson… thus the quark changes flavor. Beta decay is governed by the weak nuclear force. The electron does not have enough energy to escape the pull of the positively charged nucleus (protons), so the electron must quantum mechanically tunnel out of the nucleus, that is it borrows energy to “jump” outside the nucleus then returns the energy and travels away from the nucleus with its initial energy. The emission of the electron is also accompanied by the emission of anti-electron-neutrino. Negative beta decay is given by the following equation:

(8)

$${}_{0}^{1}n \rightarrow {}_{1}^{1}p + {}_{-1}^{0}e + \bar{\nu}_e$$

Negative beta decay is the process by which a proton decays into a neutron and a positron and emits a electron neutrino. The mass of the neutron is greater than the mass of the proton. Therefore negative beta decay does not happen without an input of energy, the binding energy is lower in the original nucleus.

(9)

$$energy + {}_{1}^{1}p \rightarrow {}_{0}^{1}n + {}_{+1}^{0}e + \nu_e$$

Beta particles are emitted at close to the speed of light and thus have a much great penetration ability and therefore more hazardous to humans, they have less ionizing energy than alpha particles.

Unlike gamma decay beta particles are given how in a spectrum of energies. For example given a stationary nucleus (zero momentum and zero kinetic energy) that undergoes beta decay. In order to conserve momentum and energy the beta particle and the nucleus must both move away from one another. However some of the energy and momentum is shared with a neutrino, the energy is given out randomly to the 3 particles, only obeying conservation of energy and momentum (conservation of mass-energy). Thus the energy of the beta particle is not discrete or characteristic of the nucleus.

State the radioactive decay law as an exponential function and define the decay constant.

Derive the relationship between the decay constant and half-life.

Radioactive decay is a random process, however if large numbers of nuclei are involved patterns, trends or averages can be found. It is found that the number of decays reduce exponentially with time. The half-life is the amount of time for half of the original unstable nuclei to decay. If there is originally N0 radioactive nuclei then after one half-life , there will be ½N0 after two half-lives there will be ¼N0. The number of nuclei left is described by the following equation:

(10)

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{\frac{1}{2}}}} = N_0 2^{\frac{-t}{T_{\frac{1}{2}}}}$$

Doing a little math:

(11)

$$\frac{N}{N_0} = 2^{\frac{-t}{T_{\frac{1}{2}}}}$$

$$log_2\left(\frac{N}{N_0}\right) = \frac{-t}{T_{\frac{1}{2}}}$$ (12)

Then changing base:

(13)

$$\frac{\ln\left(\frac{N}{N_0}\right)}{\ln(2)} = \frac{-t}{T_{\frac{1}{2}}}$$

(14)

$$\ln\left(\frac{N}{N_0}\right) = \frac{-t\ln(2)}{T_{\frac{1}{2}}}$$

(15)

$$N = N_0 e^{\frac{-t\ \ln(2)}{T_{\frac{1}{2}}}}$$

Where we now define the following as the decay constant:

(16)

$$\lambda = \frac{\ln(2)}{T_{\frac{1}{2}}} = \frac{0.693}{T_{\frac{1}{2}}}$$

Therefore the equation for radioactive decay is:

(17)

$N = N0\, e^\wedge - t\lambda$

This can also be rewritten in terms of the activity:

(18)

$A = A0\, e^\wedge - t\lambda$

Where A is the number of decays per second and A0 is the number of decays per second at t = 0.

Solve problems using radioactive decay law.

Example:

Plutonium - 239 has a half life of approximately 24,400 years. Suppose a sample of Nuclei and an initial activity of 4 mCi. Where 1 Curie (Ci) is the activity of a radioactive material that decays at the rate of disintegrations per second. -Tippens, 6th Edition, page 880

a) After 73,200 years how many of the nuclei are left?
b) What is the activity of the source after 73,200 years?

Solution

Gee I hope they're all that hard.

Outline methods for measuring the half-life of an isotope.

Students should know the principles of measurement for both long and short half-lives.

If you want to measure the half-life of substance the method is to stick a sample in front of a detector and count the number of decays. Over a period of time the number of decays will decrease and thus a plot of decays vs. time will produce a nice exponential curve. The half-life can be calculated from the decay curve. However a problem arises if the half-life is very short or very long.

Long half-lives: If the activity of a sample is so small it may be difficult to measure a significant number of decays in order to generate a decay curve. If a large amount of radioactive substance is used then a significant number of decays will occur per unit time, the activity can be measured. The decays can then be detected. The detectors are never 100% efficient, so the efficiency of the detector must be found using a source of known activity. The activity is given by the following equation:

(23)
$A = \Delta N \Delta t = -\lambda N$

Where ΔN is the change in the number of radioactive nuclei, is the change in time, N is the number of radioactive nuclei in the sample and is the decay constant which is related to the half life and thus what we want to measure. So the half-life is:

(24)
$T 1 2 = \ln(2) \lambda$

(25)
$T 1 2 = \ln(2) - (A \cdot N)$

Short half-lives: Some nuclei have such sort half-lives that transporting the sample to a detector is virtually impossible, i.e. the substance decays before you get it to the detector. In such cases the sample must be created (Artificial Transmutation) in or very near a detector. (Gee, I wonder if someone got a Nobel Prize of thinking of that solution?)

Source: http://ibphysicsstuff.wikidot.com/nuclear-physics