Models Based on Transport Phenomena

Keywords: Models, Transport Phenomena Based, Multiple Gradient

MULTIPLE GRADIENT DESCRIPTIONS

It incorporates less detailed information about the internal features of the system of interest than does the microscopic description. The forms of the equations for this model correspond to the microscopic transport equations but with modified coefficients. These coefficients are empirical and must be determined for each type of equipment or unit of interest. The essential feature of the multiple gradient descriptions is that one or more dispersion terms are important and must be retained in the model with or without the convective terms. The application of this description are to processes with turbulent flow unpacked beds or porous media.

Derivation of a Multiple Gradient Mathematical model:

Consider a square tube in which a fluid with a given solute concentration is flowing turbulently at constant temperature. Suppose the inlet concentration is now suddenly changed to a new value. We wish to derive a mathematical expression that will enable us to predict how the effluent concentration will change as a function of time. This type of situation occurs in ion exchange processes.

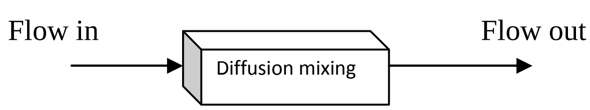

Fig 8.1: Continuous Flow of Fluid in a Rectangular Pipe

A preliminary analysis indicates tat a multiple gradient description of the system is reasonable and fairly representative description of the process. The continuity equation for the solute concentration of component I in rectangular coordinates is:

$$\frac{\partial C_i}{\partial t} + \frac{\partial v_x C_i}{\partial x} + \frac{\partial v_y C_i}{\partial y} + \frac{\partial v_z C_i}{\partial z} = \frac{\partial\left(\rho D_{ix} \frac{\partial C_i/\rho}{\partial x}\right)}{\partial x} + \frac{\partial\left(\rho D_{iy} \frac{\partial C_i/\rho}{\partial y}\right)}{\partial y} + \frac{\partial\left(\rho D_{iz} \frac{\partial C_i/\rho}{\partial z}\right)}{\partial z} + Ri \qquad \ldots 8.1$$

Assumptions

1. There are no chemical reactions so $R_i = 0$

2. If the inlet concentrations are uniform across the cross-section of the duct only gradients in the axial direction are important and the z and y derivatives of concentration can be ignored. Also consider that D_{iz} is independent of position so $D_{iz} = D_L$

3. Density can be taken to be constant. The continuity equation for the total mass is found by adding the continuity equations for all the species and is given as;

$$\frac{\partial \rho}{\partial t} + \frac{\partial v_x \rho}{\partial x} + \frac{\partial v_y \rho}{\partial y} + \frac{\partial v_z \rho}{\partial z} = 0 \qquad \text{...8.2}$$

if $\rho = constant$

$$\frac{\partial v_x \rho}{\partial x} + \frac{\partial v_y \rho}{\partial y} + \frac{\partial v_z \rho}{\partial z} = 0 \qquad \text{...8.3}$$

$$\frac{\partial C_i}{\partial t} + v_z \frac{\partial C_i}{\partial z} = D_L \frac{\partial^2 C_i}{\partial z^2} \qquad \text{...8.4}$$

Source:
http://nptel.ac.in/courses/103107096/19