Macroscopic Description

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In this we ignore all the inside details of the subsystems, and a consequence no special gradients are involved in the mathematical statements. Only time remains as a differential independent variable in the general balances. The dependent variables such as concentration and temperature are not the functions of positions and here represents averages over the volume of the subsystems. For process with clearly defined boundaries, the macroscopic equations are as follows;

Mass Balance for α^{th} species:

$$\frac{d(m_{x,tot})}{dt} = -\Delta(\rho_\alpha \langle v \rangle S) + W_i^m + r_{\alpha,av} V_{tot} \qquad \ldots 10.1$$

Momentum Balance:

$$\frac{d(\rho_{t,tot})}{dt} = -\Delta(\rho\langle v^2 \rangle S_i + \langle P \rangle S_i) + F_i^m + m_{tot} g_i + F_i \qquad \ldots 10.2$$

Energy Balance:

$$\frac{dE_{tot}}{dt} = -\Delta\left[\left(\hat{H} + \frac{1}{2}\frac{\langle v^3 \rangle}{\langle v \rangle} + \hat{\varphi}\right)(\rho\langle v \rangle S)\right] + q - W + Q^m + S_R \qquad \ldots 10.3$$

MODEL OF FILLING OF AN EMPTY CYLINDER

Assume an insulated cylinder is completely evacuated and the filled with helium from a very large source at 1 atmosphere. Set up the governing relations between the amount of gas entering and its temperature in the cylinder.

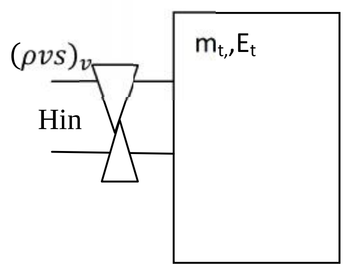

Fig. 10.1: Filing of a Gas Cylinder

We will make use of the energy and mass balances with the following simplifications;

$m_t = total\ mass$

E_t = total energy

Assumptions:

- No exit stream $(\rho V_S)_{OUT} = 0$
- No chemical reaction ($r_{xt} = 0, s_r = 0$)
- No potential energy change ($\Delta\Phi = 0$)
- No interphase mass transfer ($w^m = \beta^{(m)} = 0$)
- No work done since tank is rigid (w =0)
- No heat transfer (Q =0)

$$\frac{dM_t}{dt} = (\rho V_S)_{in} \qquad \ldots 10.4$$

$$\frac{dE_t}{dt} = [(\hat{U} + p\,\tilde{v} + \tilde{k})\rho vs] \qquad \ldots 10.5$$

Since no information is given to find the k.e. , we shall neglect this term, although the inlet nozzle may be of such a design that there is substantial k.e. in the inlet stream.

Let

$\hat{U} + p\,\widehat{\hat{v}} = \widehat{h}$, enthalpy per hour of entering stream

$E_T = U_T$ no contributions kinetic and potential energy

If we assume $c_{v,t}$ is constant and take reference temp. T = 0,

$$c_{v,t}\frac{d(Tm_t)}{dt} = [\widehat{\hat{H}}\text{Pvs}] = (\ C_P\ \text{T}\ \rho VS) \qquad \ldots 10.6$$

After differentiation of the left hand side of the above equation we have............

$$c_{v,t}\ [m_t\frac{dT_t}{dt} + T_t\ \frac{dm_t}{dt}] = c_{v,t}\,[\ c_{v,t}\ [n_t\frac{dT_t}{dt} + T_{t(pvs)in}\ ] \qquad \ldots 10.7$$

If the physical parameters are known then above equations can be solved simultaneously.

Source:
http://nptel.ac.in/courses/103107096/16