Experimental Measurement of Age Distribution Function

Keywords: RTD, Age Distribution Functions

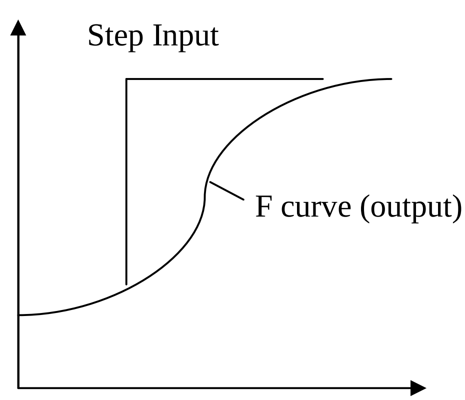
Experimental Measurement of Age Distribution Function

The experimental determination of the age distribution functions is accomplished for a particular vessel by a stimulus response technique using some sort of tracer material in the inlet fluid stream. The injection is the stimulus and the response is the tracer concentration measured in the outlet stream. The tracer can be a radioactive compound, a colored dye, and electrically conducting salt solution or another material depending on particular situation. It should have the same density, viscosity and other properties as the measure of mixing. Tracer is injected into the inlet stream in some known fashion, such as a step or sudden jump, a pulse, a sine wave other cyclic signal or even a random signal with known properties.

Danckwerts introduced the notation that the dimensionless response to a step injection of tracer be called the F curve and the dimensionless response for an impulse injection be called the C curve.

Suppose that with no tracer initially present a step function (in time) of tracer is introduced into the fluid entering a vessel. Then the dimensionless concentration-time curve for the tracer in the fluid stream leaving the vessel i.e. the F-curve shown in the following figure. F curve rises from 0 to 1.

$$F = \frac{C_\theta}{C_0}$$



$$\theta = \frac{t}{\bar{t}}$$

Fig. 13.1: F-Curve

Similarly, suppose an instantaneous pulse or shot of tracer is injected into the entering stream. The dimensionless response of time curve C is as shown in fig. With this choice of variables the area under the C curve is always unity.

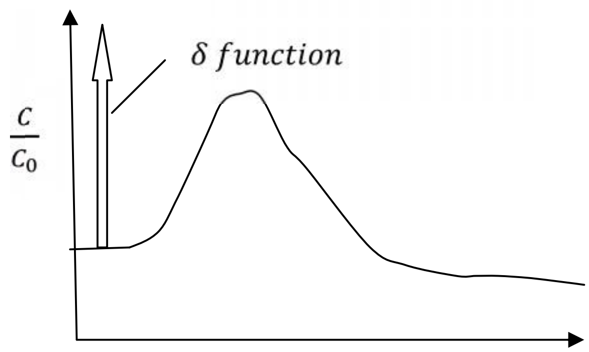


Fig. 13.2: C-Curve

$$\theta = \frac{t}{\bar{t}}$$

$$\int_0^\infty C(\theta)\, d\theta = \int_0^\infty \frac{C(\theta)}{C_0}\, d\theta = 1 \qquad \ldots 13.1$$

$$C^0 = \int_0^\infty C\, d\theta = \frac{1}{\bar{t}} \int_0^\infty C(t)\, dt \qquad \ldots 13.2$$

For a closed vessel there is a simple relationship between the E and C curve and the I and F curve. A closed vessel is defined as one in which there is no back diffusion of any sort at the entrance and exit. Most experimental setups approximately fulfill this requirement since the inlet and outlet pipes are frequently much smaller than the vessel and also the bulk flow is very much larger than any diffusion flux.

Suppose at t=0 an impulse is injected. All tracer elements of fluid have the same starting time for their ages. Thus the outlet concentration –time or C-curve is also a record of the age function of fluid element that entered the vessel at t=0 and left it at t=t, which is precisely the same as E curve.

$$C(\theta) = E(\theta) = \bar{t}E(t) \qquad \ldots 13.3$$

In other words, by injecting a tracer in pulse into fluid flowing into a vessel and determining the ages of fluid element in the output by measuring the tracer concentration in the output stream.

Age Balance:

Amount of Tracer remaining in vessel = Amount of tracer not leaving vessel

or $V\, I(t) = Q\, [1 - F(\theta)]$ …13.4

or $I(\theta) = 1 - F(\theta)$ …13.5

Comparing eq. 13.5 with $I(\theta) = \bar{t}I(t) = \int_t^\infty E(t')\, dt = 1 - \int_0^\infty E\left(t'\right) dt' d\theta$

$$F(\theta) = \int_0^t E\left(t'\right) dt' \qquad \ldots 13.6$$

= Fraction of material in exit stream younger than age t

$$F(\theta) = \int_0^t C(\theta) \frac{dt'}{\bar{t}} = \int_0^\theta C(\theta')\, d\theta' \qquad \ldots 13.7$$

$$C(\theta) = \frac{dF(\theta)}{d\theta} \qquad \ldots 13.8$$

Age Distribution Functions- Perfect Mixing and Plug Flow

Perfect mixing assumes that the vessel contents are perfectly homogenous and have the same composition as the exit stream. If we consider a step input to a perfectly mixed vessel, a macroscopic material balance gives,

$$\bar{t}\frac{dc}{dt} + c = c_0 \qquad \ldots 13.9$$

$$\frac{c}{c_0} = 1 - e^{-t/\bar{t}} = F(\theta) = 1 - e^{-\theta} \qquad \ldots 13.10$$

Consequently,

$$I(t) = \frac{1}{\bar{t}}[1 - F(\theta)] = \frac{1}{t} e^{-t/\bar{t}}$$

$$I(\theta) = e^{-\theta} \quad \text{...13.11}$$

$$E(t) = \frac{dF(\theta)}{dt} = \frac{1}{t} e^{-t/\bar{t}} \quad \text{...13.12}$$

$$I(\theta) = E(\theta) = e^{-\theta} \quad \text{...13.13}$$

In this case $I(\theta)$ and $E(\theta)$ are identical since the fluid within a perfectly mixed vessel has the same composition as that of exit fluid.

The intensity function can be found as follows;

$$\Lambda(t) = \frac{1}{\bar{t}} \frac{E(t)}{I(t)} = \frac{1}{t} \quad \text{...13.14}$$

$$\Lambda(\theta) = 1 \quad \text{...13.15}$$

In plug flow, all material passes through the vessel without any mixing; each fluid element stays in the vessel for exactly the same length of time. For a step input the front or interface between the tracer and non tracer fluids travel down the vessel and come out of the other end in a time equal to main residence time. This $F(\theta)$ curve is a step input curve.

$$F(\theta) = U(t - \bar{t}) \quad \text{...13.16}$$

Where, $U(t - \bar{t}) = 0, t < \bar{t}$

$= 1, t > \bar{t}$

Then,

$$I(t) = \frac{1}{\bar{t}} [1 - F(\theta)] = \frac{1}{t} [1 - u(t - \bar{t})] \quad \text{...13.17}$$

$$or\ E(t) = \frac{dF(\theta)}{dt} = \frac{dU(t-\bar{t})}{dt} = \delta(t - \bar{t}) \quad \text{...13.18}$$

$$I(\theta) = 1 - u(\theta - t) \quad \text{...13.19}$$

$$E(\theta) = \delta(\theta - 1) \quad \text{...13.20}$$

The intensity function for plug flow is,

$$\Lambda(t) = \frac{\delta(t-\bar{t})}{1-U(t-\bar{t})} \quad \text{...13.21}$$

$$\Lambda(t) = 0, 0 \le t < \bar{t} \quad \Big\} \quad \text{...13.22}$$

$$= \quad \infty, t = \bar{t}$$

$$\left.\begin{aligned} \Lambda(\theta) &= 0, 0 \leq \theta < 1 \\ &= \quad \infty, \theta = 1 \end{aligned}\right\} \qquad \ldots 13.23$$

Source:
http://nptel.ac.in/courses/103107096/13