Analysis and Modeling of Evaporators Using Newton Raphsonôs Method (Without Boiling Point Rise)

Keywords: Multiple Effect Evaporator, Newton Raphson's Method

The Newton raphson method consist of the repeated use of linear terms of Taylor series expansions of the functions $f_1, f_2, f_3, f_4, f_5, f_6$.

$$0 = f_j + \frac{\partial f_j}{\partial V}\Delta V_o + \frac{\partial f_j}{\partial T}\Delta T_1 + \frac{\partial f_j}{\partial L}\Delta L_1 + \frac{\partial f_j}{\partial T}\Delta T_2 + \frac{\partial f_j}{\partial L}\Delta L_2 + \frac{\partial f_j}{\partial A}\Delta A$$

Where (j = 1,2,3,4......6)

$\Delta V_0 = V_{k+1} - V_k$;

$\Delta T_1 = T_{k+1} - T_k$;

$\Delta T_2 = T_{2,k+1} - T_{2,k}$

$\Delta L_2 = L_{2,k+1} - L_k$;

$\Delta A = A_{K+1} - A_k$;

Where the subscripts k and k+1 denotes the k th and k +Ist trials

These six equations may be stated in compact form by means of the following matrix equation

$$J_k \Delta X_k = -f_k$$

Where J_k is called Jacobin matrix and

$\Delta X_k = X_{k+1} - X_k = [\Delta V_0 \Delta T_1 \Delta T_2 \Delta L_2 \Delta A]^T$

The subscripts k and k+1denotes that the elements of the matrices carrying these subscripts are those given by k and k+1st trials, respectively. In the interest of simplicity k is omitted from the elements of X_k, J_k, f_k. On the basis of set assumed values for the elements of column vector X, which may be stated as transpose of the corresponding row vector.

$X_k = [V_0 T_1 L_1 T_2 L_2 A]$

The corresponding values of elements of J and f are computed.

A display of the elements of J_k and f_k follows-:

J_k

$\partial f_1/\partial V$	$\partial f_1/\partial T$	$\partial f_1/\partial L$	$\partial f_1/\partial T$	$\partial f_1/\partial L$	$\partial f_1/\partial A$
$\partial f_2/\partial V$	$\partial f_2/\partial T$	$\partial f_2/\partial L$	$\partial f_2/\partial T$	$\partial f_2/\partial L$	$\partial f_2/\partial A$
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	-	-	-
$\partial f_6/\partial V$	$\partial f_6/\partial T$	$\partial f_6/\partial L$	$\partial f_6/\partial T$	$\partial f_6/\partial L$	$\partial f_6/\partial A$

$f_k = [\ f_1, f_2, f_3, f_4, f_5, f_6\ ]^T$

The functions $f_1, f_2, f_3, f_4, f_5, f_6$ and their partial derivative which appear in J_k are continuous and the determinant of j should is not equal to zero, then only Newton raphson method will converge, provided a set of assumed values of the variables which are closed enough to the solution can be found.

Let us consider a triple effect evaporator with forward feed arrangement without boiling point elevation. The functions f_1 f_2 f_3 f_4 f_5 f_6 obtained are stated below-

1 Effect-

Enthalpy balance-

$f_1 = F(h_f - h_1) + V_0 \lambda_0 - (F - L_1)\lambda_1$

Heat transfer rate-

$f_2 = UA(T_0 - T) - V_0 \lambda_0$

2 Effect-

Enthalpy balance-

$f_3 = L_1(h_1 - h_2) + (F-L)\lambda_1 - (L_1 - L_2)\lambda_2$

Heat transfer rate-

$f4 = U2\ A(T_1 - T_2) - (F - L_1)\lambda_1$

3 Effect-

Enthalpy balance-

$f_5 = L_2 (h_2 - h_3) + (L_1 - L_2) \lambda_2 - (L_2 - L_3) \lambda_3$

Heat transfer rate-

$f_6 = U_3 A (T_2 - T_3) - (L_1 - L_2) \lambda_2$

If the changes in the specific heat with temperature in the neighbourhood of the solution to equations are negligible, then the sensible heat terms $(h_F - h_1), (h_1 - h_2), (h_2 - h_3)$ may be replaced by the respective equivalents $C_p (T_F - T_1), C_p (T_1 - T_2)$ and $C(T_2 - T_3)$.If the variation of the latent heats with temperature Are also regarded as negligible in the neighbourhood of the solution, then

J_k =

$-\lambda_0$	$-FC_p$	λ_1	0	0	0
$-\lambda_0$	$-U_1A$	0	0	0	$U_1(T_0-T_1)$
0	L_1C_p	b_{33}	$-L_1C_p$	λ_2	0
0	U_2A	λ_1	$-U_2A$	0	$U_2(T_1-T_2)$
0	0	$-\lambda_2$	L_2C_p	b_{55}	0
0	0	$-\lambda_2$	U_3A	λ_2	$U_3(T_2-T3)$

Where $b_{33} = C_p(T_1 - T_2) - (\lambda_1 + \lambda_2)$

$b_{55} = C_p(T_2 - T_3) - (\lambda_2 + \lambda_3)$

23.1 Example:

It is desired to design a triple effect evaporator system to concentrate the solute from a 10% solution to 50% by wt. The feed rate is 50,000lb/hr and it enters the first effect liquid as liquid at 100^0f. Forward feed is to be used. Saturated vapour of the solvent at 250^0F is available for satisfying the heating requirement for the first effect. The third effect is to be operated at an absolute pressure corresponding to a boiling point of 125^0f for the pure solvent. Neglect boiling point elevation as well as the variation of heat capacities and latent heat of vaporisation with temperature and composition. Determine the area per effect the temperature T_1 and T_2 and the flow rates and the composition and the rate.

Given- $C_p = 1$ btu/lb ^{o}f

$\lambda_{20} = \lambda_1 = \lambda_3 = 1000$ btu/ lb^{o}f

$U_1 = 500$, $U_2=300$, $U_3=200$

Solution:

The calculation procedure may be based initatedon the basis of reasonable set of assumptions,

1. T_0-T_1 =42^0f, T_1-T_2 =42^0f, T_2-T_3 =41^0f

2 .Solvent evaporated in first effect =14,000lb/hr

Solvent evaporated in second effect =14,000lb/hr

Solvent evaporated in third effect =12,000 lb/hr

3.A =1000 ft for each effect

4.V_0 = 15000/hr

A scaling procedure is used to reduce to reduce the magnitude of the terms appearing in the functional group and matrices. For computational purposes, it is desirable to have tems with magnitude near unity. The following scaling procedure easy used.

1. Each functional group is divided by the product $F\lambda_0$and new functional group so obtained is denoted by g_j.
2. All flow rates were expressed as afraction of the feed rate F that is L_j =l_jF and V_j =v_jF
3. All temperature temperatures were expressed as function of steam temperature as follow: T_j=$u_j$$T_0$
4. The area of each effect is expressed as a fration of term proportional to the feed rate in the following manner:A_j= a_j(F/50)

After this scaling procedure has been applied to the functional expressions, thematrices J, ΔX and f take the following forms-

ΔX_k=[Δv_0 Δu_1 Δl_1 Δu_2 Δl_2 Δa]^T

f_k =[g_1 g_2 g_3 g_4 g_5 g_6]^T

j_k =

-1	b_{12}	λ_1/λ_0	0	0	0
-1	b_{22}	0	0	0	b_{26}
0	b_{33}	b_{33}	b_{34}	λ_2/λ_0	0
0	b_{42}	λ_1/λ_0	b_{44}	0	b_{45}
0	0	$-\lambda_2/\lambda$	b_{54}	b_{55}	0
0	0	$-\lambda_2/\lambda$	b_{64}	λ_2/λ_0	b_{65}

Where the elements of J consist of the partial derivative of the g‘s with respect to new set of variables.

$b_{12} = -C_pT_0/\lambda_0$; $b_{34} = -l_1C_pT_o/\lambda_0$; $b_{22} = -U_1aT_0/50\lambda_0$; $b_{44} = U_2aT_0/50\lambda_0$; ; $b_{32} = -l_1C_pT_0/\lambda_0$; $b_{54} = -l_2C_pT_0/\lambda_0$; $b_{42} = -U_2aT_0/50\lambda_0$; $b_{64} = -U_2aT_0/50\lambda_0$; $b_{26}=-[U_1(1-u_1)T_0]/50\lambda_0$; $b_{33} =[C_p(u_1-u_2)T_0-(\lambda_1+\lambda_2)]/\lambda_0$; $b_{46}=[U_2(u_1-u_2)T_0]/50\lambda_0]$;$b_{66}=[U_3(u_2-u_3)T_0]/50\lambda_0$
; $b_{55}=[C_p(u_2-u_3)T_0-(\lambda_2+\lambda_3)]/\lambda_0$

Then on the basis of the assumed values of the variables, the matrices J_0,f_0 and X_0 are used in the first trial follow-:

J_0 =

1	-.25	1	0	0	0
-1	-2.5	0	0	0	.42
0	.18	-1.958	-.18	1	0
0	1.5	1	-1.5	0	.252
0	0	1	.11	-1.959	0
0	0	-1	1	1	.614

f_0=[-.088 .120 .030 -.028 .058 -.116]^T

X_0=[.300 .832 .720 .664 .440 1.000]^T

Convergence to within about six significant numbers was achieved in three trials. The values of X complicated at the end of first four trials are displayed.

Table No. 23.1: Calculate Value of Unknown Variables

Trial no	v_0	u_1	l_1	u_2	l_2	A
1	.359374	.879493	.760498	.743069	.494740	1.13835
2	.357773	.874144	.760762	.733878	.494848	1.13703
3	.357771	.874138	.760763	.733868	.494848	1.13703
4	.357771	.874138	.760763	.733868	.494848	1.13703

Thus the desired solution is-:

$V_0 =v_0F$ =17,888.5lb/hr; $T1=u_1T_0$ =183.467F;$L_1=l_1F$=38,038.1lb/hr;L_2-l_2F=24,742.4 lb/hr;

A-(aF/50) =1,137.03lb/hr; L3 =FX/x_3=10,000 lb/hr;$x_1=FX/L_1$ =.131447; $X=FX/L_2$=.202082

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